THE MATHEMATICS TEACHER

Volume XXXIV Number 2

Edited by William David Reeve

Program, National Council of Teachers of Mathematics

Hotel Chelsea, Atlantic City, N. J., February 21–22, 1941

Theme: Mathematics in a Defense Program

Friday, February 21, 1941.

8:00 A.M. Meeting Board of Directors

10:00 A.M. *Junior High School Section*

Presiding: F. Lynwood Wren, George Peabody College for Teachers, Nashville, Tennessee.
The Significance of the National Arithmetic Committee Report for Teachers of Junior High School Mathematics—L. J. Brueckner, University of Minnesota, Minneapolis, Minnesota.
Algebra and the Ways of Democracy—Hildegarde Beck, McMichael Intermediate School, Detroit, Michigan.
Relational Thinking for Competency in Junior High School Mathematics—H. C. Christofferson, Miami University, Oxford, Ohio.
Discussion.

10:00 A.M. *Training Prospective Teachers.*

Presiding: Virgil S. Mallory, State Teachers College, Montclair, New Jersey.
Mathematical Foundation Necessary for High School Teaching—Howard H. Fehr, State Teachers College, Montclair, New Jersey.
Preparation of Teachers of Secondary Mathematics—H. T. Karnes, Louisiana State University, Baton Rouge, Louisiana.
Training for the Teaching of the Appreciative Values of Mathematics—L. S. Shively, Ball State Teachers College, Muncie, Indiana.
Discussion.

10:00 A.M. *Multi-sensory Aids—Theory.*

Presiding: Mary A. Potter, Supervisor of Mathematics, Racine, Wisconsin.
Tools and How They can be Used in Teaching Plane Geometry—Robert C. Yates, Louisiana State University, University, La.
Materials and Techniques of Making Mathematical Models—E. F. Assmus, Nutley High School, Nutley, New Jersey.
The History of Elementary Mathematical Instruments—Edwin W. Schreiber, Western Illinois State Teachers College, Macomb, Illinois.
Employing Visual Aids to Teach Locus in Plane Geometry—Edna H. Young, Rutherford High School, Rutherford, New Jersey.
Discussion.

12:00. Luncheon, Delegates and State Representatives.

1:15 P.M. *Multi-sensory Aids—Movies.*

1:15 The Worker's Old-Age and Survivors Insurance Account.
1:35 Individual Differences in Arithmetic.
1:55 Geometry in Action.
2:10 Social Security Benefits.

2:30 P.M. *Arithmetic Section.*

Presiding: Foster Grossnickle, Jersey City, New Jersey.
What Measures Do Children Know, and Why?—Mabel Cassell, Emmanuel Missionary College, Berrien Springs, Michigan.
How the Lincoln School Uses its Activity Program to Teach Arithmetic—John R. Clark, Lincoln School of Teachers College, Columbia University.
Problem Solving in Arithmetic—Thomas J. Durell, Assistant Commissioner of Education at Trenton, New Jersey.

2:30 P.M. *Senior High School Section.*

Presiding: Dorothy Wheeler, Bulkeley High School, Hartford, Connecticut.
On the Remodeling of Old Algebra Problems—Maurice Hartung, University of Chicago, Chicago, Illinois.
Arithmetic Maintenance Program in the Senior High School—Myron F. RossKopf, John Burroughs School, Webster Groves, Missouri.
Mathematics and Aviation—Carl M. Shuster, State Teachers College, Trenton, New Jersey.

4:00 Tea. Members of the Council are guests of the New Jersey Association of Mathematics Teachers.

7:30 P.M. *General Meeting.*

Presiding: Mary A. Potter, Supervisor of Mathematics, Racine, Wisconsin.
Address of Welcome
Mary C. Rogers, President Association of Mathematics Teachers of New Jersey, Westfield, N. J.
Response
F. Lynwood Wren, George Peabody College for Teachers, Nashville, Tenn.
Mathematics in CCC Education—G. M. Ruch, U. S. Office of Education, Washington, D. C.
Work of the War Preparedness Committee of the Society and The Association—Marston Morse, Institute for Advanced Study, Princeton, New Jersey.

Saturday, February 22, 1941

8:30 A.M. Business Meeting.

9:30 A.M. *Senior High School Section.*

Presiding: William David Reeve, Teachers College, Columbia University, New York City.
An Entirely New Demonstrative Geometry for Schools, A New Approach and a New Departure—Ralph Beatley, Harvard University, Cambridge, Massachusetts.
Advocates of other novel proposals concerning demonstrative geometry, whether these novelties be psychological or mathematical, are especially invited to take part in the discussion for which ample opportunity will be provided.

9:30 A.M. *Training Teachers in Service.*

Presiding: Carl N. Shuster, State Teachers College, Trenton, New Jersey.
The High School Principal Looks at the Teacher in Service—Frank J. McMakin, Principal William L. Dickinson High School, Jersey City, New Jersey.

The Role of the Mathematics Teacher in Our Defense Program—C. R. Atherton, Hershey Junior College, Hershey, Pennsylvania.

Helping Teachers to Analyze Their Problems—Rolland R. Smith, Central High School, Springfield, Massachusetts.

Discussion.

9:30 A.M. *Multi-sensory Aids—Theory.*

Presiding: E. H. C. Hildebrandt, State Teachers College, Montclair, New Jersey.

Visual Aids—Available Equipment and Principles of Use—M. Richard Dickter, Furness Junior High School, Philadelphia, Pennsylvania.

Motion Pictures for the Teaching of Mathematics—Henry W. Syer, Culver Military Academy, Culver, Indiana.

Mathematics in Three Dimensions—John T. Rule, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Discussion.

12:00 Discussion Luncheon.

It is *most important* to make luncheon reservations in advance stating first, second, and third choice of tables. Luncheon tickets are $1.40 per plate and reservations should be made with Dr. Howard Fehr, State Teachers College, Montclair, New Jersey..

Presiding: Maurice Hartung, University of Chicago, Chicago, Illinois.

Tables	Leaders and Topics
1.	Charles M. Austin: Algebra, a universal language
2.	William Betz: The message of our new national reports in their relation to the present mathematical situation.
3.	I. S. Carroll: Some mathematics problems taught in defense classes.
4.	H. C. Christofferson: Meanings in mathematics.
5.	Marie Gugle: The supervisor visits the classroom.
6.	Maurice Hartung: What topics should be discussed in department teachers meetings?
7.	Martha Hildebrandt: For those who can't or don't.
8.	Ruth Lane: The ninth grade curriculum.
9.	Nathan Lazar: Some untapped possibilities in Plane Geometry.
10.	Jonas T. May: Problem solving in Algebra.
11.	A. Brown Miller: Junior high school subject matter for junior high school pupils—regardless.
12.	Gordon Mirick: Selected topics in mechanics suitable for inclusion in the high school course in mathematics.
13.	Joseph B. Orleans: The reading technique in the teaching of mathematics.
14.	Nanette Roche: Enrichment in mathematics in the Baltimore junior high schools.
15.	John T. Rule: Can Geometry be used to solve problems in Algebra?
16.	Vera Sanford: What are the vital things in high school mathematics?
17.	W. S. Schlauch: Financing the cost of national defense.
18.	Edwin W. Schreiber: Why the defense program needs mathematics.
19.	Veryl Schult: Defense of Arithmetic in the senior high school.
20.	Joseph Shuttlesworth: Can the second course in Algebra be made more attractive and educationally more valuable?
21.	Rolland R. Smith: Developers vs. explainers.
22.	C. Newton Stokes: Disciplining the whole individual—the part played by mathematics.
23.	Ruth Stokes: In a defense program, what mathematics and how much shall we make available to all competent students in the secondary schools?
24.	Helen Walker: What statistical ideas essential to understanding current social phenomena can be taught successfully to high school students?
25.	Dorothy Wheeler: Mathematics in Connecticut's defense program.
26.	H. W. Charlesworth: Mathematics exhibits for high schools.
27.	Robert Yates: Rules and tools.

28. Hildegards Beck: Junior high school mathematics and teaching.
29. Walter W. Hart: Should there be a two-lane highway in secondary mathematics?
30. Virgil S. Mallory: Configurations of problems in junior high school mathematics.

2:30 P.M. *Arithmetic Section.*

Presiding: C. Louis Thiele, Supervisor of Exact Sciences, Detroit, Michigan.
Presentation and Discussion of the New Arithmetic Yearbook—(Sixteenth) of the National Council of Teachers of Mathematics.
Presentation of the Sixteenth Yearbook—Robert L. Morton, Chairman of the National Council Committee on Arithmetic, Ohio University, Athens, Ohio.
The Administrator Examines the Sixteenth Yearbook—Stanley H. Ralfe, Superintendent of Schools, Newark, New Jersey.
The Reaction of the Psychologist to the New Arithmetic Yearbook—Frederick S. Breed, University of Chicago, Chicago, Illinois.
Discussion.

2:30 P.M. *Junior High School Section.*

Presiding: Mary Rogers, Roosevelt Junior High School, Westfield, New Jersey.
Organizing Mathematical Instruction around Mathematical Topics vs. Organizing It through Varieties of Experience—E. H. C. Hildebrandt, State Teachers College, Montclair, New Jersey.
Arithmetic, What Is Its Place in Junior High School?—John R. Clark, Principal Lincoln School of Teachers College, Columbia University, New York City.
The Unsolved Problem of Ninth-Grade Mathematics—A Study and Appraisal of Present Trends—William Betz, Rochester, New York.
Discussion.

2:30 P.M. *Secondary School Section.* (Speakers from Private Schools)

Presiding: Donald E. Mac Cormick, William Penn Charter School, Germantown, Pennsylvania.
What I'd Like to See Done in Secondary Mathematics—Mrs. Helen MacDonald Simmons, Winsor School, Boston, Massachusetts.
Don't Forget the Good Student!—E. P. Northrop, Hotchkiss School, Lakeville, Connecticut.
There will be ample opportunity for discussion from the floor.

4:00 P.M. *Visual Aids—Slides and Films.*

I. Film Slides

1. Geometry in Nature
2. Geometry in the Home
3. Geometric Solids in Nature and Architecture
4. The History of the Measurement of Length
5. Is Seeing Always Believing?

II. British Mathematical Films

1. Rate of Change
2. Theorem of Pythagoras
3. A Hypocyclic Motion
4. Angle Sum of a Triangle
5. Equation $\ddot{X}+X=0$
6. The Generation of Involute Gear Teeth
7. Resultant Circle and Straight Line
8. Resultant Ellipses
9. Hypocycloid Gear
10. Levers

4:00 P.M. Meeting Board of Directors.

6:30 P.M. Annual Banquet.

Address: Clarity Is Not Enough—Professor Lancelot Hogben, University of Aberdeen, Scotland.

Professor Hogben is now lecturing at the University of Wisconsin on genetics and will return to the University of Aberdeen later.

Local Arrangements Committee

Mary C. Rogers, Westfield, Chairman
Committee members are chairmen of other committees
Ferdinant Kertes, Perth Amboy, Publicity
Marian Lukens, Camden, Hospitality and Reception
Howard F. Fehr, Montclair, Luncheon
J. Dwight Daugherty, Paterson, Banquet
B. W. Lidell, Atlantic City, Hotel Arrangements and Equipment, Registration
Fred Bedford, Jersey City, Exhibit

Program Committees

Secondary Schools: Ruth Lane, Iowa City, Iowa, Chairman, Ralph Beatley, Cambridge, Mass., John R. Clark, New York City, Edith Woolsey, Minneapolis, Minnesota.

Arithmetic: C. Louis Thiele, Detroit, Michigan, Chairman, Foster Grossnickle, Jersey City, New Jersey, Clifford B. Upton, New York City.

Teacher Training: L. H. Whitcraft, Muncie, Indiana, Chairman, G. H. Jamison Kirksville, Missouri, Virgil S. Mallory, Montclair, New Jersey.

MEMBERSHIP IN THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS BY STATES, DECEMBER 1940

Alabama	59	Maryland	84	Pennsylvania	362
Arizona	12	Massachusetts	244	Rhode Island	25
Arkansas	36	Michigan	211	South Carolina	79
California	275	Minnesota	115	South Dakota	21
Colorado	95	Mississippi	47	Tennessee	82
Connecticut	89	Missouri	154	Texas	177
Delaware	13	Montana	24	Utah	13
Washington, D.C.	98	Nebraska	66	Vermont	18
Florida	59	Nevada	7	Virginia	87
Georgia	52	New Hampshire	18	Washington	76
Idaho	5	New Jersey	226	West Virginia	47
Illinois	553	New Mexico	16	Wisconsin	205
Indiana	152	New York	618	Wyoming	23
Iowa	134	North Carolina	86		——
Kansas	169	North Dakota	19	United States Total	5568
Kentucky	79	Ohio	310	Foreign	178
Louisiana	38	Oklahoma	116		——
Maine	26	Oregon	50	Grand Total	5783

What Is Happening to College-Preparatory Mathematics?*

By H. M. Bacon
Stanford University

A serious problem confronts teachers of mathematics in preparing students for an engineering education.

A few weeks ago some of you may have read a rather astonishing article in the *Saturday Evening Post* entitled, "Lollypops Versus Learning," written by a high school teacher. In it the author points out that a great many public school administrators have attempted to put into practice what they believe to be the ideas and methods of Progressive Education without having anything approaching a clear understanding of what these ideas actually involve. The result has been, in many cases, not only ludicrous to the observer, but detrimental, if not disastrous, to the unsuspecting youth of the community. The situation has frequently been caricatured in some of our best humor magazines, and I imagine that most of us have laughed at the cartoon showing the youngsters in the most "advanced" type of schoolroom. Some are boxing, others wrestling, some hammering on the furniture, some beating drums, some skipping rope—all but one solemn little boy who sits gloomily aloof in a corner. One of the other boys is seen calling to him, "You'd better find an *outlet*, or you'll catch hell!". Or we have chuckled at another lad in the midst of a similar schoolroom diligently writing on the portable blackboard $2+2=4$. The teacher is gently admonishing him, "Come, come, Johnnie; stop fooling around!"

Now I have no intention of quarrelling with my Progressive Education friends. There is much of value in their proposals which if properly understood and handled could produce excellent results. But, we must realize that much effort is wasted, and serious deficiencies in training of pupils are developing where teachers and administrators put too much faith in the fine words which describe certain *methods* of teachings, and forget *what* it is that is to be taught. "Projects," "activity programs," "teaching in terms of real life situations," etc., etc., are all very well, but talking about such things doesn't prepare boys for an engineering school, or, in fact, for very much of anything. I am reminded of the answer given to a certain question propounded in a questionnaire sent to the English Faculty of a certain western university. The question was this, "Would a student do himself any good in the examination by indicating words which he would look up the spelling of (sic), in a 'real life situation'?" The answer given by the English Department was, "Taking an examination *is* a real life situation."

After all, we must face reality. Young men and women are coming out of public high schools, and we expect that they will possess a modicum of training in certain fundamental college preparatory subjects. In this connection, I want to point out two trends in the planning of mathematics curricula in the secondary schools which must be recognized as of primary concern to all of us who are interested in the promotion of sound engineering education. The first is a noticeable trend towards dilution of the subject matter of the present mathematics courses (or perhaps I should say, of the mathematics program which has been familiar to us, since it is *not* the present program in a good many places). The second is the trend to postpone the offering of elementary algebra to the tenth

* A paper read at the Luncheon Meeting of the Mathematics Section of the Society for the Promotion of Engineering Education at Berkeley, California, on June 25, 1940.

year, a movement with which is closely connected the "stepping up" or postponement of much arithmetical instruction to the later grades of the elementary school. The two trends are, naturally enough, closely allied.

Before going further with these remarks I want to state most emphatically that I am concerned with the mathematics program as applied to students in the high school who have real aptitude for mathematics, or who need a full course in mathematics to prepare them for later college work. And may I add that this includes others besides those who are sure that they are going to study engineering and other science courses in college. I have no thought of defending the forcing of any particular subject upon an inept or properly uninterested pupil. I realize, as we all do, that not everybody will profit by four years of college preparatory mathematics. But the unusually capable and gifted students must not be neglected. Let me quote from the letter of a high school mathematics teacher in one of our San Joaquin Valley towns, "If there is any group in our high schools today that is neglected, it is the group made up of gifted pupils." There is no doubt that, in the past, the high school course has been geared to the requirements of the college preparatory pupils. Other pupils, who will make just as good citizens, have been neglected. Recently there has been a great change in emphasis, and the latter group has come into its own. In fact, it seems to have monopolized the scene. The dull students, who must not be permitted to suffer the sense of frustration which follows failure, have, at least in some cases, set the standard of accomplishment demanded. The loss of opportunity to the capable students seems to have escaped attention. While this is certainly not universally true, still it is a tendency which must not be overlooked. I am sure that we can all agree upon the necessity of making available adequate courses in mathematics for those who can and who wish to pursue them.

Let me speak first, very briefly, of the dilution of subject matter which seems to be taking place in the making of the mathematics curriculum. There is quite a strong movement to "step up" arithmetic subjects to later and later grades—apparently largely on the theory that the older children are, the more easily they learn and retain information and skills. As one result there were eighteen classes in remedial arithmetic at one nearby city high school a year ago. There seems to be a desire on the part of some curriculum makers to drop many of the usual topics in algebra, partly because they are difficult. Let me quote from a note which appeared in THE MATHEMATICS TEACHER for May, 1940. This was written by a member of the staff of University High School in Urbana, Illinois, and a leader in the educational world. He says, "The ways of the traditional mathematics texts do indeed pass understanding. Factoring is still retained in practically all the current Algebra books even though this phase of work is obsolete and almost without any practical or stimulating value for the students . . . " He proposes to introduce instead a process known as "Alligation" which appeared in earlier texts and with which I am not familiar. Comment upon this statement seems superfluous.

Let me quote a proposal made by a gentleman who is a professor of chemistry in a mid-western university, and who should know better, "Science teachers use the decimal system in practically all of the measurements and mathematical calculations they make. I do not see any real reason why pupils should continue to add fractions by the common denominator method. Why not change the fractions to decimals? It is much easier, and I believe with a little practice it could be done more rapidly. All the pupils would need to do is learn the decimal equivalents of a few common fractions. How much simpler subtracting, multiplying, and dividing would become if decimals were used." While there is no danger of such a fantastic proposal making any headway, it is clear that attention must be given by those interested in sound arithmetical instruction to problems of curriculum planning. Those responsible are anxious to hear the voice of experience. It is our duty to add our bit to this voice, and to be sure that the advice is good, and not like that of the zealous advocate of the decimal representation of fractions. People who are in a position to give counsel should give good counsel.

I have spoken of a movement to "step up" arithmetic subjects in the elementary school. This has its counterpart and its sequel in a movement which is becoming more and more widespread in the high schools of the country. I refer to the postponing of elementary algebra from the ninth to the tenth grade. This leaves plane geometry for the eleventh grade, and any further algebra, solid geometry, and trigonometry to the twelfth graae, which is the last year of the usual high school course. Most of us probably think of the high school mathematics program as consisting of one year of algebra in the ninth grade, one year of geometry in the tenth grade, both of these being required for college entrance. These are normally followed by one year of algebra and one year devoted to solid geometry and trigonometry, these last two years being optional, even for college preparatory students. The new "three-year plan" contemplates replacing the first year of algebra with a course in what is termed "general mathematics." This promises to be, in some cases at least, a hodge-podge of alleged applications to "real life situations" of arithmetic, algebra, and geometry of the most elementary sort. While these topics are to be arranged in a more or less systematic order, the student has time to become acquainted with any one in only a perfunctory way. This course is to be required of all students, and afterwards algebra, geometry, and trigonometry follow in the last three years if the student so elects or the university insists. This scheme may, or may not, be a good one for the average student. It is certainly bad for the superior student who must waste his time in the ninth grade, and who is cut short in his study of algebra by the restriction of time. The ninth grade course in "general mathematics" may take various forms. In one Southern California high school (I quote the description) "approximately the first semester is given to review and remedial work in arithmetic situations to build up arithmetic background both for those who later take algebra and for those for whom this is a terminal mathematics course. . . . The second semester is devoted to what is generally classified as 'consumer' mathematics, with special attention to local problems." Let me quote a paragraph from an article by John W. Studebaker, United States Commissioner of Education. "It is trite to remark that prediction is hazardous. In this instance, however, certain present tendencies will almost certainly continue. The progressive education movement, so far as it concerns mathematics, will undoubtedly continue to accumulate facts which suggest activity units for the integration of mathematical processes around social rather than mathematical topics. This will be particularly true in the junior high school (that is in ninth grade) where the point of view will be that of the mathematics of the *consumer* in ever increasing degrees. Instead of such topical sequences as rectangles, angles, circles, the Rule of Pythagoras, square root, etc., we shall find an organization based upon such topics as ways of earning a living, budgeting the family income, buying on the installment plan, planning a savings program, etc. Within these general spheres or centers of home, community, and national activity will be found units or activities of genuine social significance."

It is impossible to go into the arguments advanced in favor of, or against, this plan of deferring algebra. It is, however, pertinent to observe that while some of the advocates of the three-year plan are mathematics teachers, many of the proponents among school administrators have had a background of experience which included no mathematics beyond the minimum required for graduation from high school. I have no quarrel with the program, provided it does not deprive the qualified student of a chance to take a good solid four years of high school mathematics. That it sometimes does is demonstrated by the experience of the daughter of a friend of mine. She was in the ninth grade of a junior high school, and her parents wanted her to take algebra. She was particularly apt in mathematics, and, although she was not expecting to be an engineering student in college, she wanted very much to take algebra. But she simply was *not permitted* to begin the work.

That the movement is not inconsequential is clear from an article by Frank Lindsay of the California State Department of Education which appeared in *California Schools* in April of 1939. He reports that, at that time, replies from 324 schools established the fact that more than half of those institutions had moved algebra from the ninth to the tenth grade. The author says, "Plane geometry is an eleventh-year subject in more than a third of these schools; in some it is postponed to the twelfth year. By review and remedial procedures 54 per cent of the schools attempt to secure mastery of fundamental processes." The director of the Secondary Curriculum Section of the Los Angeles School System reports that for the past three years the postponement of algebra has been a curriculum policy of the secondary schools of that city, and that the movement is becoming quite general throughout much of the country.

In conclusion, I should like to call your attention to the fact that the two California Sections of the Mathematical Association of America and also the America Mathematical Society have found the situation of such interest and importance that they have passed, and given some publicity to, a resolution concerning the matter. I should like to read to you a letter which was sent out to a large list of school administrators in this state.

The officers of the Northern California Section of the Mathematical Association of America believe that you will be interested in the following resolution, passed by unanimous vote at the regular annual meeting of the Section held in Berkeley on January 27, 1940. Similar resolutions were passed unanimously by the Southern California Section of the Mathematical Association of American at its meeting on March 2 at Compton Junior College, and by the American Mathematical Society at its meeting on April 6, at Berkeley.

"The Northern California Section of the Mathematical Association of America and those in attendance at the meeting of the Section wish to go on record as favoring that a program of mathematics be provided in the secondary schools, beginning normally with algebra in the ninth year, to be available for those who wish to elect it or who otherwise need it in preparation for college work. It is felt that a capable student should be able to secure solid geometry and trigonometry in the secondary school."

This resolution in no way implies that college preparatory courses should be required of all students. But these three organizations feel strongly the importance of continuing to provide substantial courses in mathematics for those who need them in preparation for later work or for those who choose to elect them. We believe that, to be effective, these courses must begin with algebra in the ninth year.

We know that you will give this problem the serious consideration which it merits, and hope that you will call this resolution to the attention of those members of your school system concerned with the teaching of mathematics.

Northern California Section,
MATHEMATICAL ASSOCIATION OF AMERICA
H. M. Bacon
Secretary-Treasurer.
(Assistant Professor of Mathematics, Stanford University.)

The few replies received indicate that the resolution has, at least, not been unnoticed. I may add that not less than two groups of California mathematics teachers have passed similar resolutions.

But resolutions by mathematics teachers do little good, I fear. We too obviously appear to have an axe to grind. It is pre-eminently to the engineers that we look for aid. They are practical men of affairs who work in "real life situations" as some of our educators would say, and their voice will be heeded where ours is not. Furthermore, the problem is theirs too. It is notorious that the average engineering student in college is now crowded to the limit of his time and energy, that his program is oftener far heavier than that of his fellow students. If it becomes necessary, as seems likely, to add to his already over-loaded study program most of high school mathematics, either he will be compelled to spend an extra year in college, or the standards of his engineering training will suffer grave relaxation.

What action you will care to take is, of course, for you to decide. But some action seems imperative.

IN CONNECTION with the above article by Professor Bacon the reader should consult the note about "The Doctrine of Postponement" on page 324 of *The Mathematics Teacher* for November 1940.—EDITOR.

Traders and Trappers

By Carl Denbow
Ohio University, Athens, Ohio

This is the story of an episode that occurred before the French and Indian War, when the French fur traders were driven out of North America, and before many of the Indians had firearms. A missionary named Martin had, with his family, lived with the Black Bear Indians and converted them to Christianity, and had taught them to speak English. Then, one night, a band of Wolf Indians had attacked and massacred many of the Black Bear tribe, and also the missionary and his wife. Their three small sons escaped: Allan, aged 3; Ralph, 5; and Oliver, 6. They were adopted into the Black Bear tribe, and were well-liked by everyone. When our play opens, 10 years later, they were already noted for skill with arms, and quickness of mind.

The action occurs in a forest, on the edge of an Indian village. In the background is a tent, against which leans a bow. There are four crude stools in the foreground, and leaning against a tree is a large slate, on which, as the play opens, Oliver is drawing a map showing a river, a hill, and an Indian village. Ralph is watching him.

The three boys are tanned and healthy. Each carries a large hunting knife in his belt. Allan is the most impetuous of the three. All of them are of courageous bearing, which reflects itself in their speech.

Oliver: (Pointing to the map he has drawn on the slate) Here, Ralph, is the village of the Wolf tribe. We haven't had a fight with them for nearly two moons now.

Ralph: If only we had half as many men as they! We'd show them!

Allan: (Entering) Hello, there you are! (closer, and confidentially) There's something strange going on in this village. I don't know just what it is, but I've a hunch that traveller, Jonathan, who came a few days ago, would gladly knife us all in the back.

Oliver: Oh, no, Allan! He's a missionary, just like Dad. What have you noticed that's strange?

Allan: Well, er, nothing exactly, but I trust my hunches.

Ralph: Look, boys, here comes the chief! (The Chief enters. He is a tall, dignified man, with a pleasant smile which he uses sparingly.)

Chief: Boys, I would like to have a long talk with you. (pauses) I don't need to tell you how hard we all worked this winter, trapping for furs. Do you know what we wanted them for?

Ralph: Yes, Sir, I do! We're going to trade them to the French for guns!

Chief: (smiling) Yes, Ralph. The Wolf tribe outnumbers us seven to one. The French Chief from way across the big water told his traders here to give us guns, because we are Christians, just like the French are. They don't want the Wolves to burn our village, and kill us. But (pauses), the Trader here is afraid of the Wolf tribe. One of our scouts learned he intends to cheat us in our big trade, and give us so few guns that we can't defend ourselves. The Wolves have scared him into doing that.

Oliver: (angrily) We won't let him!

Chief: That is what I wanted to talk to you about.

Allan: What can we do?

Chief: (pauses) Before your father came here, the Black Bear were as ignorant as the other Indians. Your father was not only the bravest man I have ever met, he was the wisest. He taught us all to speak English, how to care for our wounded, and many other things. Do you know what is the most important thing he taught us?

All: What?

Chief: He taught us how to *count*. To be sure, we had words in our Indian language for one, two, three, four, and five; we called them eina, koopla, sona, vort, and vint. We had no words, no names, for six, seven, or any other numbers.

Oliver: Why didn't you just make up words for the other numbers?

Chief: We didn't know that there were any *others*. I know it seems strange now, for now we know that we can name as many numbers as we like. All we need is words for them. But don't think we were *too* dumb. The Wolf Indians can, even now, count only to three, and when I asked your father about it, he said that from the rising sun to the setting sun (he points, with a sweeping gesture) only the most learned and the most civilized tribes can count past ten.

Ralph: Why is counting so important?

Chief: You are still a little young to ask that question, Ralph, but since you are very quick, I shall tell you. Without counting we could not save the right amount of food to last through the winter, with a little to spare. Without counting, it would be very hard to divide up the fruits of the hunt fairly. (interrupted by the entrance of Jonathan, the travelling missionary. As he enters, Allan walks behind him and points toward him, to tell Oliver and Ralph; "this is the man I distrust.")

Chief: Welcome, Jonathan, I was just thinking about you. The Black Bear tribe has a hard job to do. I was just now going to ask the boys here to help us with it, and perhaps you would help us, also. (Boys look surprised.)

Jonathan: (hesitating, looking scared) Well, I'm not much of a fighting man, and I'm hardly strong enough for hard work, but, er, what is it you want?

Chief: (appears surprised, and disappointed, as if he had thought very highly of Jonathan) No, it's nothing like that. I'll explain what we need. I was just telling our boys, here, that one of the valuable things their father taught us was the ability to count. He taught us the *names* for the different numbers, different names than the ones we had used before. But the French have a way of making little marks for numbers, instead of writing out the words, and that is why they can figure so much better than we can, and cheat us in the trades. To protect ourselves against the Wolf Indians we need guns, and for them we must trade furs, and so you can see that the most important problem facing our tribe is that of learning some system of using marks for numbers. The French guard their secret carefully. Our best spy tried for weeks to discover it, but he was captured. (Proudly) He gave his life for the tribe.

Ralph: (eagerly) I know two of the marks they use. (He goes to the slate: all numbers not written out in *words* in the dialogue from here on are to be written on the slate by the various speakers, or pointed at, if they are already on the slate. Portions of the map already drawn on the slate are erased to make room for the figures.) They just make a scratch for "one" like we do, "1." But for "two" they don't make two scratches, as we do, 11, but they use a mark like this: "2".

Oliver: I remember that, now that you mention it, and also the mark they use for "three." They write "3" instead of our 1 1 1. But I don't remember any more. Maybe I'll think of them. I didn't realize how important numbers are.

Chief: (pleased) Friends, you have a good start. I must go now, to the council fire. I am trusting you boys to work out a system for us. I believe white men are better at that than we are, and we are lucky to have you with us. (Leaves.)

Jonathan: (Speaks quickly, anxious to persuade the boys to his point of view.) I'm very glad for this chance to help our Indian friends. I believe I can help you with your adventure of creating a new number system. First, don't you think the Indian way of writing two scratches for the number two, like this (writes 11) is better than using this mark you mentioned, "2"? It's more logical. If one mark like this "1" means one, then two marks such as "11" should mean two. That's the natural, God-given way. These godless French, inventing a mark like "2"! Let's just follow the old system. For three we'll use three scratches, and so on.

Allan: (Whispers, but everyone hears him.) I still don't trust this man! All this talk about the God-given way!

Allan: (aloud, to Jonathan) Do you mean that Christians use scratches, as you want us to do, but heathens don't?

Jonathan: Yes, that is right.

Allan: On our father's watch the hours are marked with a funny kind of mark, one mark "I" for one, likewise "II" and "III" for two and three, but at four it has this mark "IV," and so on. My father didn't like the system and said some *heathen* Romans started it. Heathen, mind you! (angrily, to Jonathan) So I think you're trying to fool us!

Oliver: (shocked) Allan, you musn't talk like that. This man is a missionary, like our own father, and he's trying to help us. I think we should take his advice. (turning to Jonathan) Please forgive my brother's rudeness.

Jonathan: (smiling maliciously) Of course. Now, for these numbers—

Allan: (interrupting) I remember the mark the French traders use for four! It's this! (He writes "4.") Let's just make up marks of our own for the rest of the numbers. There's no sense in the way this man (he points toward Jonathan) suggests, of having five scratches for the number five, and so on. It gets too complicated. (To Jonathan) If you are really trying to help us, why don't you tell us the French system? You have been in the big cities of the white men, New York and Boston, and you must know the system!

Oliver: Yes, Allan is right. Tell us what system is used, by the French and by our own Americans out East.

Jonathan: (starts to walk away, then whispers loudly, as if thinking aloud) If I don't tell them, they won't trust me any more. (He says, aloud.) I'll tell you, and then you can see how Godless and unnatural it is. They use this mark "4" for the number four, as Allan says, then this "5" for five, "6" for six, "7" for seven, "8" for eight, and "9" for nine. You can see what funny marks those are. They are hard to learn, hard to use. God never intended that we should use them. He gave us fingers so that we could use one finger to mean one, two fingers for two (illustrates by holding up his fingers), and so on, just like the scratches we make on a marking stick. (pauses) Well, boys, I must be leaving now, as I am journeying to another tribe today. Good-bye! (Leaves.)

Boys: Good-bye!

Allan: I'm going to have him trailed. I *still* don't trust him! (Leaves.)

Oliver: You know, I think Allan has something there. Why should we make five scratches to mean the number five, when we can use a mark like this 5? We could use the Roman numbers Allan mentioned, like those on the watch, using V for five, and so on, but why not start at the beginning and have a brand new mark for *each* number, like these that Jonathan showed us (points to the 1, 2, 3, . . . , 8, 9 on the slate), not just a new mark, V, for five?

Ralph: These numbers are fascinating, don't you think? It's interesting that the Indians used one kind of numbers, the white men another, and those Romans, whoever *they* were, used a third kind. It makes me wonder how many kinds of numbers there are in the whole world.

Oliver: Everyone really uses the *same* numbers, I think. Take five, for example. We now call it "five" and write it 5 for short, while our tribe used to call it "vint" and wrote it 11111. These Romans wrote it V. It's still the same number, though. It seems to me that there is really only one set of numbers but many different ways to write them, and our problem is to invent the easiest marks for writing them.

Ralph: Yes, you are right. All we are looking for, after all, is the easiest way to use marks for numbers, and so far we've succeeded up to nine. That's strange! Johnathan stopped at nine. (points to slate) He didn't tell us what the French traders use for ten. But surely they don't go on forever, inventing new marks for every new number! It would be too hard to *remember* all of them. I wonder if Jonathan stopped at nine so he wouldn't give away their secret.

Oliver: I see what you mean, Ralph. The French can't go on inventing new marks forever. But they can't start repeating, either, because numbers *don't* repeat. No two numbers are the same. I wonder what they do? (pauses, and thinks.) I remember, when Mother started to teach me how to write, before her death, I practiced on a sentence with a 23 and a 19 in it. (He pronounces these "two-three" and "one-nine" and writes them) I've forgotten what little reading and writing she taught me, but I have kept the papers I wrote on, and I saw them the other day, with those figures on them.

Ralph: The two-three must mean two, and three more, that is five; then (excitedly) the one-nine is just what we want, because one and nine more is ten! (Points to the 19 on the slate).

Oliver: I don't think so. They wouldn't write two-three (he writes 23) if they meant five, they'd write 5. Still, one-nine seems like as good a way as any to write ten, *if* we aren't going to invent a *new* mark.

Ralph: Wouldn't three-seven also mean ten, in the same way, because three and seven more is ten? (He writes 37, and then 28, and 82.) And wouldn't two-eight and also eight-two mean ten? Won't that be awfully confusing?

Oliver: (slowly, thinking hard) Yes, that's *not* a very good way to write ten, after all. We seem to be stuck. We can't go on inventing new marks forever, and when we try a combination of these, (pointing to the list on the slate) we get into an awful mess. (louder) I've got it! We'll do both!

Ralph: How can we do *both?*

Oliver: We'll use a *new* mark, σ or Δ or something, for ten. Let's use this: Λ. (Writes these three marks, then erases the first two.) Then this: Λ1 will mean ten and one more, or eleven. Λ2 (reads this "ten-two") will mean ten and two more, or twelve.

Ralph: That will work clear up to nineteen. Then what?

Oliver: (eagerly) Then we'll use 2Λ to mean two *times* ten, that is, twenty. 2Λ1 will mean twenty and one more, and so on. Now we are getting someplace!

Ralph: Then 2Λ means twenty, while Λ2 means ten and two, or twelve. Can we do that?

Allan: (comes rushing in, with the Chief, and an Indian who has Jonathan's arms tied, and is dragging him.) Look what I found! It's just as I suspected. We followed Jonathan nearly a mile. In a clump of trees and bushes he met the French trader and started bragging to him that he had fooled us, that we were on the wrong path now with our number system. He's not a missionary at all, but a spy!

Oliver: Allan, you were right! Jonathan certainly had me fooled. He's a scoundrel! The French are trying hard to cheat us, but we'll win out in spite of them! (pauses) Allan, let me show you what we have done, while you were gone. We've invented a new mark for ten, Λ, then for eleven we write Λ1, for twelve Λ2, and so on. Then when we come to twenty we put the 2 in front of the Λ, and write 2Λ. Ralph doesn't think it's right to have the 2 in front of the Λ mean two *times* ten, and when the Λ comes before the 2 (points to Λ2) have it mean ten *plus* two. What do you think?

Allan: I think it is a good idea. Otherwise we would have to invent so many new marks we couldn't remember them all.

Jonathan: Boys, I *have* been a spy, and I'm heartily ashamed of myself. Let me prove it to you by telling you the truth about the white man's numbers. (Adds, under his breath: maybe you'll be easy on me then.) Oliver and Ralph here have performed one of the most astounding feats of all history! *Of course* it's all right to have 2Λ mean two *times* ten, while Λ2 means ten *plus* two. It took a thousand years for our ancestors to discover that! But once they did, they progressed so rapidly that they are now far ahead of the Indians, who haven't discovered it. So far ahead that they make rifles by using number principles! (pauses) There are only two slight improvements which have been made on the system you boys have discovered.

Boys: (happy) What are they?

Jonathan: Men learned, after many, many years, that no new mark for ten is needed. Instead of writing twenty-one this way (he writes 2Λ1), that is, two times ten plus one, and instead of writing 2Λ3 for twenty-three, they learned that they could leave out the Λ and just write 21 and 23. That makes the system harder to learn, beceuse the ten is not written, but it makes computation *so* much easier! A small child can now add numbers that would have stumped the wisest men of old!

Oliver: Then the *two* in 23 means "two times ten" while the *three* means "and three more"! No wonder we couldn't figure out what the two-three on my early writing-lesson meant! It's so easy when someone tells us, and so hard to figure out alone!

Ralph: We wrote thirteen as Λ3, in our system. How do you write it, if you have no mark for ten?

Jonathan: That's the interesting thing about the white man's system. Just as twenty-three is two times ten plus three, written simply as 23, so thirteen is just *one* times ten plus three, and we just write it 13.

Ralph and Oliver: (eagerly) I see!

Jonathan: The way we write ten is the strangest part of the whole system. Just as twelve is one times ten with *two* added, so ten, itself, is one times ten, with *nothing* added. So, instead of inventing a mark for ten as you did, our ancestors invented this mark, zero (he writes 0) to mean *no* arrows, or *no* enemies, just as 3 means three arrows, or 3 enemies. Then they wrote ten this way (he writes 10) meaning one times ten and *no* more. So you see we don't need a special mark like yours to mean ten, if we have a mark for zero. (pauses) Boys, I want to make a confession to you. When the Trader told me your tribe was trying to discover his number system, or invent another one just as good, I laughed at him. I told him that the greatest scientists and thinkers of Greece and Rome and other nations had never discovered a good system. Only the Hindus had learned the secret, and how they hit upon it, nobody knows. So I'm here to say that you boys can be proud of this day's work. All of civilization is based on a convenient way of writing numbers and here, alone, in the depths of the forest, you have invented such a way! Believe me, I'm sincere when I say, regardless of what happens to me, I'm proud to have known you.

Oliver: (proudly, to the Chief) I think we have finished the job you set for us.

Chief: (has been standing quietly, with folded arms, since entering) Friends, I have always thought that only white men could understand numbers, but this system is so simple that I can learn it easily. Boys, the gratitude of our tribe to you is as boundless as the sky, and will endure while the sun rises and sets. The French will not cheat us out of rifles again, and we need fear no enemies, great or small. (to Jonathan) Come along. The tribal council will decide your fate. (to the boys) We want you there also. You will be the youngest men ever to sit on our council, the youngest warriors our tribe has ever boasted.

(*Curtain*)

What Mathematical Knowledge and Abilities for the Teacher of Geometry Should the Teacher Training Program Provide in Fields Other than Geometry?*

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Before one ventures to assert what mathematical knowledge *any* teacher of *any* mathematics should have, it is fitting that he remind himself—and others, possibly—that "what knowledge" is only half the problem: Equally arresting is the question of how that knowledge is to be acquired. The knowledge and abilities listed in this paper are necessary, but not sufficient if received as a "hand out."

Every teacher of teachers-to-be has upon his shoulders the responsibility for setting the stage so that mathematical knowledge *happens* to his students. This stage-setting includes, over and over again presenting examples among which some significant likeness appears; impelling students to classify the examples; recognizing the resulting dawn of a generalization when it comes; evoking careful and precise statements of these new inductive inferences; carrying out an experimental test of each newly-discovered—or newly-suspected—theorem by a search for exceptions. Then, and only then, should come the final satisfaction of deductive proof, to slip the new theorem into its niche in the system; and lastly, of course, must come some opportunity to recognize and to make some applications.

Most of us, when we learned mathematics in college courses, had only these last two steps: proofs of theorems, and practice in application; even the latter often was slighted. Knowledge so acquired makes a mere tool, not only of mathematics, but also of the learner. For teachers whose pupils are to grow in power, there is no substitute for approaching mathematical knowledge through discovery. Those of us who try to help with teacher training cannot remind ourselves too frequently that "Teachers teach as they are taught, not as they are taught to teach."

1. *The Nature of Measurement.* First on this list is a knowledge of the nature of measurement. This can be taught best in the mathematics of the elementary and secondary schools. But if neglected there, it should be worked into the college arithmetic and trigonometry courses; and if a man appear in calculus without it, a "time out" there is mandatory. The longer it is postponed, the more difficult it becomes. A college senior or graduate student doing practice teaching in geometry and not possessing this understanding of measurement, finds it most difficult; for by that time he knows so much that isn't so!

A knowledge of the nature of measurement begins with comprehension of the differences between the questions "How many?" and "How much?" The choice of a unit in terms of which to describe a distance, for example, is a tremendous step—the step that makes possible the description of that magnitude in terms of number. Attendant upon this are two more realizations of great import: direct measurement is a process of actual superposition; hence, all numbers that result from actual measurement are approximate.

With the measurement of area, and the choice of a suitable unit for it, comes the next great step; no further progress is possible without a consideration of form: "What shape of figure shall we superpose

* A paper read at the St. Louis meeting of the National Council of Teachers of Mathematics in February 1940.

upon the floor of the room in order to describe its size?" A teacher who has not (in his own early experience) faced this question before learning that "length times width equals area," has probably missed the significance of the transition from direct to indirect measurement—and so may his pupils! With such knowledge the teacher can be more convincing when he shows that measurement numbers of surfaces can be obtained indirectly by computations based on direct measurement numbers of distances. Man's attempts to prove the rules for these computations for ideal figures, are the story of demonstrative geometry; were it not that straight line and circle forms can be described precisely only in terms of number, plane geometry would not be a branch of mathematics.

Furthermore, the geometry teacher who does not possess this fundamental concept of the nature of measurement, contentedly ignores the fact that indirect measurement involves computation with approximate numbers. That brings us to the second item on this list:

2. *Approximate Computation.* The teacher of geometry should know how to judge accuracy of results computed from approximate numbers. To ignore the effects of errors upon products, quotients, and roots is like hiding one's eyes and stopping one's ears in a thunder storm; these effects are as inescapable as the elements.

High-school pupils, bless them, will not take rules for approximate computation on authority. The teacher must help them to set up an experiment that will reveal the best rule to follow. For example, they may choose two numbers, as $2\frac{1}{7}$ and $351\frac{8}{11}$ find their product first in common fraction form. Then they can convert both factors and product to decimal form and multiply approximations involving first few, then more significant figures, each time comparing the result with the true product. These comparisons readily reveal the most sensible rules to follow in multiplication.

The next three items, in this list of knowledge are closely related to the last:

3. *Numerical Approximations and Irrational Numbers.* The teacher of geometry should know the difference between an irrational number and a repeating decimal.

4. *Proof that Irrational Numbers Exist.* He should be able to prove that at least one irrational number exists.

5. *Irrational Measurement Numbers in Theoretical Geometry.* He should be able to set the stage for having pupils discover that irrational numbers arise inevitably in the measurement of straight line figures—even the perfect imaginary ones for which the theorems are proved.

A correspondence between rational numbers and repeating decimals involves sufficient knowledge of geometric progressions to find the rational number represented by any given repeating decimal.

The proof that at least one irrational number exists, though a topic in functions of a real variable, is a simple *reductio ad absurdum* proof, easily accessible to students on the secondary school level. The teacher, however, to have an illuminating point of view, should know enough of functions of a real variable to appreciate Dedekind's and Cantor's proofs that rational and irrational numbers are the only kinds of numbers needed for measuring distance on a line.

That irrational numbers arise in the measurement of plane geometry figures, can be shown in an attempt to measure with the same unit the side and the diagonal of a square. Then it is only one step further to see that if a proof about number relations in geometric figures is to hold for all cases, it must hold for irrational numbers—numbers which, whether we like it or not, can be written only approximately.

The question, "In just what mathematics course should all these topics appear?" is not within the scope of this paper. Perhaps a course in fundamental concepts might include them. In at least one teachers college they are taught by the supervisor of student teaching in semi-weekly conferences with her student teachers, and she thinks that's a bit late in the program.

Nevertheless, a prospective teacher who hasn't met these ideas in his own secondary school work, should meet them in college somewhere, before he expands the sad circle by teaching geometry himself.

6. *Algebra.* Every teacher of geometry should be agile in translating words into algebra. For two thousand years many proofs of demonstrative geometry have remained clumsy and cumbersome for want of being expressed in the language of algebra—all this because the proofs were first written by a scholar who had no algebraic symbolism at his command. The teacher of geometry should know his algebra as a way of saying things. At least a part of its symbolism should have been re-invented by himself. This invention *could* have happened—and *can* happen—in the classroom (if not on the end of Mark Hopkins' log!) in situations created by some good teacher. That teachers of geometry do not even label figures—angles, lines, arcs, and the like—with symbols easily adaptable to algebraic statements, is evidence of the availability of their algebra.

Though this translation of statements about straight line and circle figures into algebra is the most vital contribution of algebra to secondary school geometry, there are certain other special topics which must also appear in algebra courses for geometry teachers; for example, a development of the laws of exponents which will illuminate the theory of logarithms from the start. Every teacher of geometry should be comfortable with logarithms before becoming mechanical with them; else, he will be tempted to thrust them upon *his* pupils, too, instead of revealing them to his classes. Mention has already been made of the need for a knowledge of geometric progressions, especially for finding the limit of the sum when the common ratio is less than unity. One needs, too, the sequence of experiences in algebra which leads to extensions of the number system to the field of complex numbers. Without that he is cut off from a full interpretation of such things as what happens when one tries to construct a triangle with two sides of 8 and 5 respectively with an angle of $52\frac{1}{2}$ degrees opposite the 5-unit side. Whether he presents them in his classes or not, the teacher needs algebraic interpretations of geometric phenomena for his own mathematical maturity.

7. *Trigonometry.* No one questions that trigonometry plays a part in a geometry teacher's preparation. However, it is not enough for him to be able to quote definitions and to work type problems involving trigonometric ratios. He should appreciate what theorems of plane geometry he must prove before he can prove that trigonometric ratios exist for a given angle. This point of view dictates an approach through considering the special case of an acute angle. Generalizing the definitions of sine, cosine, and tangent must follow, not precede, the acute angle case in a sound psychological approach.

This is also the point of view from which a teacher can tell at what stage in the geometry course to make the trigonometric ratios available to high-school pupils; and it is the point of view from which one can recognize the powerful use of these ratios for convenience and brevity in proofs of many theorems, especially those involving similarity.

Every geometry teacher should know the law of cosines as a theorem which includes both the Pythagorean theorem and its converse. He should understand the triangle area formula, $S=\frac{1}{2}ab \sin C$, well enough to tell how soon it can be made available to geometry pupils by proof. He should know how to reveal the redundancy in the regular polygon area formula, $P=\frac{1}{2}nsa$, where n = the number of sides, s = the number of units in one side, and a = the number of units in the apothem; he should know how to find the areas and perimeters of regular polygons of n sides inscribed in, and circumscribed about a circle of radius r; how to find ratios of those areas and perimeters, and the limits of those ratios as n increases without bound.

The geometry teacher should use tables of trigonometric ratios and logarithms understandingly, not merely mechanically. He should round off results of computations appropriately, intelligently acknowledging the limitations of his data and of the tables in all applications. And he should know how to keep his thinking straight in proving trigonometric identities. It is a rare student in some instructors' college classes who can separate "if" from "then" in his identity proofs.

So again, it must be admitted that whether or not a person's knowledge of trigonometry is dynamic in the teaching of geometry, depends largely upon the approach through which he acquired it.

8. *The Calculus.* The fundamental concepts of the calculus and an appreciation of its power are the next items on this list. Without the concept of limit and its bearing on the tratment of numbers which continuously echange together, one is unable to deal with problems in change of angles, distances, areas, and volumes. Questions about designsof cylindrical cans, boxes, etc., of maximum volume for a given surface ought to arise in solid geometry. Even though they are not answered then, the teacher should be able to direct a discussion that will stimulate curiosity and give promise of satisfaction to any student who will pursue the study of mathematics a little farther. The insight and mathematical maturity necessary for directing such a discussion are not *always* outcomes of a college course in the calculus, but they *can* be; and they certainly cannot be obtained from any other course.

9. *Logic.* Not everyone will agree that logic is a branch of mathematics, but since many college and university mathematics departments give credit for a logic course toward a mathematics major, and since without it a geometry teacher certainly cannot reveal to his pupils the grandeur and beauty of an organized system of knowledge, *therefore*, it must be admitted that a functioning knowledge of the fundamentals of logic must be a part of the geometry teacher's preparation.

He should be able to distingiuish readily between something known by authority, something known by experience, and some thing known by reasoning—that is, by deductive proof. He should appreciate the arbitrary nature of definitions sufficiently to be free from the tyranny of other people's definitions ready made. No teacher who abhors a change in definitions possesses the attitude from which to direct pupils in formulating definitions of their own —a very necessary experience if the reasoning of geometry is to transfer to other situations. Then, too, one must be forced to admit the necessity of a few undefined terms. One must comprehend the nature of assumptions and our freedom in the choice of them. He must realize the part inductive inference plays in the transformation of knowledge into power. He must realize that this power to meet new situations, this prediction of solutions to problems, is composed of deductive inferences; that all scientific prediction is deduction; that all transfer of training, in fact, in so far as "transfer of training" means "transfer of knowledge," is deduction. And the person equipped to teach should realize that deductive inferences do not spring naturally nor soundly from generalizations taken on authority. A generalization needs to happen in a pupil's mind; it should come, whenever possible, as an inductive inference. Proof by a chain of deductive inferences may follow afterward.

The teacher should sense well enough what a converse is to tell when he must say "a converse" instead of "the converse." He should recognize contrapositive statements and should know the simple *reductio ad adsurdum* proof that if one accepts a statement, he also accepts its contrapositive. He should be able to state an opposite (that is, an inverse) of a theorem, and he should know the relation of an inverse to a converse.

This list, of course, includes but a small bit of the realm of logic, but it is a necessary bit for one who is to teach geometry.

Without it, one cannot detect redundancies and inconsistencies in a system of *geometric* knowledge, to say nothing of attempting an organization of his own beliefs on politics, economics, international affairs, social issues, ethics, or religion. Neither can he diagnose the mistakes his geometry pupils make in reasoning, nor can he ask the kind of questions that evoke straight thinking from his pupils.

10. *Probability and Theory of Statistics.* One who teaches for transfer in geometry should have some insight into prediction based on statistical analysis. A person who has had no experience in discovering generalizations through classifying, tabulating, and examining observational data, and then, from his findings, trying to predict within limits what *will* happen (or to tell what *did* happen) in other cases not observed—one with no such background is likely to miss the medium through which the reasoning of geometry transfers to other subjects.

Even geometry teachers who are fairly well versed in the theory of statistics often make the mistake of considering statistical reasoning as a thing apart from the deductive reasoning of demonstrative geometry. In geometry, it is true, the deductions are based on assumptions and theorems to which no exceptions arise. They are generalizations about ideal figures, beautiful in their consistent perfection, never confronted with disconcerting contradiction by brute fact. On the other hand, when deductive reasoning is applied to the physical world, the deductions usually must be based on generalizations that hold $n \pm x$ times out of a hundred, instead of always and forever. The straight thinker must be able to judge the reliability of conclusions springing from statistical analysis. But subject to inevitable error though they are, these inferences are still induction and deduction, and are unlike those of the geometry class only in that they must be examined much more carefully for an honest estimate of the margin of error.

Teachers who comprehend this difference between reasoning about phenomena and reasoning in geometry, all too often jump to the conclusion that high-school pupils should learn to identify and use the reasoning processes through some such confusing material, rather than through the one field of subject matter which embodies the simplicity and the consistency necessary for the beginner. That field is the subject matter of Euclidean geometry: the measurement of figures made up of straight lines and circles.

11. *Familiarity with Great Fundamental Concepts.* Every teacher of geometry should possess other great fundamental concepts of mathematics, such as one-to-one correspondence, invariance, and arbitrary determination; and he should be *conscious* of possessing them.

To appreciate that the relations between lines, angles, arcs, and areas are abstract number relations, one must appreciate that for every learner, the concept of abstract number must be distilled from experiences with one-to-one correspondence.

The concept of arbitrary determination, and its bearing on the concept of personal freedom, is a necessary possession of an educated person. To live so that one retains some feeling of personal sovereignty over his own fate and yet does not dissipate his energy fretting against the inevitable, he must be able to distinguish between a situation in which he is a free agent with an infinite number of possible choices, a situation in which he is free to choose one of a finite number of possibilities, and a situation in which previous choices have closed off entirely all but one path. This concept of arbitrary determination can and should be an outcome of the study of secondary school geometry. Whether or not it *is* achieved there, depends upon whether or not the teacher is sufficiently aware of the concept to call attention to its examples.

Such questions as these illustrate the point: "How many possible plans of proof have you in this exercise? If you choose the first one you named, how many choices have you for the first step in your proof? If that is your first step, what must be the second step?" etc. Construction problems, also, present a rich variety of opportunities for this purpose: "If you are to construct a quadrilateral whose opposite angles are equal, how many choices have you for the first angle? How many choices have you for the next angle?" Frequently, some impulsive pupil will suggest constructing a triangle having its angles equal respectively to three given angles, right after he has learned to construct a triangle with the three sides given. What happens when it comes time to insert the third given angle is an experience that will make him appreciate most vividly a situation in which arbitrary determination—freedom, if you please—does not exist.

Invariance, likewise, has a contribution to make to the education of the high-school pupil—if his teacher knows how to call attention to it! No doubt there are other great mathematical concepts which would yield a significant contribution on the secondary school level, if those of us who teach teachers were just aware of them.

12. *History of Mathematics.* The last item on this list is some knowledge of the history of mathematics and of the lives of great mathematicians. While one should not expect this knowledge to be too scholarly in detail, it should be sufficient to give one some feeling for the drama of mathematical discovery and invention, and to give him an appreciation of the part mathematics has played in the development of civilization.

In conclusion, one should be reminded that this list represents one person's idea of the minimum essentials for a good geometry teacher's mathematical knowledge outside the field of geometry. The more he knows the better—unless it be all on one topic! But again let us not forget that *how* he comes to know *what* he knows is a question equally important and *much more blandly neglected in teacher training.*

Our Mathematics Classes

To learn Arithmetic
Takes work, it's not a trick
To make an A
Our problems we must state.
We learn percent and rate
By working very late
Each single day.

When we reach algebra
We feel we've traveled far
The road of Math
No student ere resigns
Bur journeys on by signs
Sometimes with tears and whines
The upward path.

Euclid has made us love
His theorems to prove
In geometry.
Hypothesis we know.
Conclusion we must show
So we can write below
Our Q.E.D.

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Improving the Teaching of High School Mathematics*

By Elizabeth Sue Dice

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The Texas Section is to be commended for beginning a long-time study of improving the teaching of mathematics in Texas. The teachers of the secondary schools welcome the opportunity to work with the college group. The teachers of the elementary schools are just as interested. The problem of improving the teaching—and the studying—of all subjects is one which should challenge the interest of parents and of teachers from the nursery schools through the graduate schools.

Four needs, from the point of view of the public school system, for the improving of the teaching of high school mathematics are the bases of this discussion.

1. *Build in both parents and pupils the following attitudes: giving lessons first place, recognizing the importance of mental age, regarding study hours as inviolate, and encouraging persistence.*

The attitudes of parents and pupils, as a factor in improving tecahing, cannot be overestimated.

a. *Giving lessons first place.* Mothers, even more than boys and girls, put dates, popularity, and paving the way for invitations to college fraternities before lessons. Parents and pupils need to evaluate immediate pleasures and to learn to weigh them against postponed pleasures.

b. *Recognizing the importance of mental age.* Experts make scientific studies of plant and of low animal life. Not in commercial laboratories but certainly in home and school conferences parents and teachers should persistently make intelligent studies of improving unborn children; of considering six-year olds with mental ages of seven or eight, and vice versa; and of recognizing gaps in mental ages caused by change of schools or of teachers, by illness, by stubborn adults who try to force children to *their* (the adults) preferred patterns, and by that most pernicious of all practices—double promotion. Frequently, ten-year olds with a mental age of twelve have, especially if double promoted, the same mental and chronological age when they become fifteen or sixteen. Mental growth is as erratic as physical growth; yet parents who do not think of boasting of a child's physical progress disregard his high school, college, and adult mental foundation in order to be able to say: "The *youngest* graduate is my son!"

Ten years ago six-year olds began to enter school and were given a course previously planned for seven-year olds. If anyone thinks one year is of no consequence, he should try teaching these pupils high-school mathematics. When they reach college, the Texas Section of the American Mathematical Association will be inviting a first-grade teacher to talk on "Improving the Teaching of First-Grade Mathematics." If the twelve-grade idea materializes for all schools, professional curriculum makers, if not watched by teachers who know that a mathematics foundation which is to stand has to be built gradually and painstakingly, will be offering college mathematics in the twelfth grade. Curriculum makers, who earn a living by catering to the public's thirst for change, are not wholly at fault. Tax payers and school officials should arrange for competent classroom teachers to have sufficient school time to serve as balance wheels for

* Given at the Texas Section of the American Mathematical Association, Southern Methodist University, Dallas, Texas, March 30, 1940. (This Section is making a study of the increasingly large percentage of failures in college freshman mathematics and, as a part of the study, asked a high school teacher to discuss the above topic.)

those who do not have to teach the courses they write. Teachers who, when there is no money for curriculum makers, use their Saturdays and Sundays to write makeshift courses of study, deserve balance-wheel recognition. Furthermore, colleges are not asking for *more* mathematics, but for *thorough* mathematics to be taught below the college level. There is already too much pushing down of mathematics into the preceding grades. Theorems arranged by Euclid for college men have receded to the secondary-school level. Small wonder that Bernard Shaw suggests that future generations may imbibe differential calculus with their mothers' milk!

c. *Regarding study hours as inviolate.* How many homes *have* study hours? If this should be made the *n*th question of the 1940 census, the answers would justify the listing of "regarding study hours as inviolate" as a needed parent-pupil attitude. Nothing worthwhile can be mastered without study; and, aside from furthering the scholarship of the individual, forming a regular-quiet-study-hours' habit will contribute incalculably to the serenity of his later living.

d. *Encouraging persistence.* This is a restless, something-for-nothing age. Adolescents, feeling the unrest and lacking guidance in self-control, either by precept or by example do not stay at a task long enough to test their ability with any degree of accuracy. In many instances adolescents lack, not ability, but persistence. All too frequently parents discourage persistence by thoughtlessly boasting that *they* "couldn't learn mathematics either."

The self-expression-for-children age began about fifteen years ago. In high school now the products of that age are expressing themselves; they study mathematics *if* they want to! A guest speaker would not think of inquiring into the self-expression of the children of college professors!

2. *Supply all grades, one to twelve inclusive, with mathematics teachers who understand and believe in mathematics; and who, given a reasonable load and sufficient time, will see that basic principles and sensible approaches recur with increasing difficulty many times.*

a. *Supply teachers who understand and believe in mathematics.* In Texas less than one-third of the high school mathematics teachers have majored in mathematics. As yet all of the remaining two-thirds have not minored in it. The recent teach-major-or-minor ruling requires fewer semester hours in mathematics for mathematics teachers than are required of other teachers in the subjects which they teach. Arithmetic teachers are just as poorly prepared. Mathematics majors who do not adjust themselves to beginners or to their surroundings and who do not inspire pupils to want to learn are also poorly prepared. Texas builds roads. If she wants to, she can build mathematics teachers,—not just a scattering few whom the school officials, not recognizing the importance of correct number concepts for beginners, invariably assign to the upper grades; but she can certificate, in grades one through twelve, only the teachers who are as excellent as our roads. One comforting thought is that principals do not always recognize, hence do not always advance the excellent teachers. Teachers already in service can, by being loyal to each other and to their group, lay the first stones for attaining such excellency in their profession. Colleges, if for no higher motive than that of self protection, should strive to lay further stones. The public, by making the teaching profession attractive to potential leaders, can experiment, perhaps economically, at least humanely, in laying stones calculated to decrease crime by education instead of by persecution.

b. *A reasonable load and sufficient time.* I purposely did not say "teaching" load. Extra-curricular, committee, and counseling assignments are disproportionate parts of the load. In school as in church, club, or any other work the conscientious person is the one who does not have a reasonable load. It is wasteful to assign a teacher five classes of forty pupils each and ten weekly extra-curricular hours. Even if doing that much well should be physically possible, no teacher is prepared to do efficiently so many different types of work. Coaches who are poor mathematics teachers are no more to be censured than school officials who put athletics before scholarship or the public who expect one man to do the work of two.

A class hour rarely has sixty minutes for mathematics. Subtract time for announcements over the loud speaker, assemblies; traffic studies; special-week observations; counseling; thrift; and selling tickets for or advertising games, dances, plays, annuals, and newspapers. Divide the remainder by spring fever plus "in the spring a young man's fancy . . . !" After-school athletics, clubs, and paper routes take precedence over lessons; thus no hour is available for individual conferences. These activities, significant factors in a well-balanced program, should have a reasonable amount of *provided* time, but not an ever-increasing part of the class or study time.

c. *See that basic principles and sensible approaches recur with increasing difficulty many times.* Note *basic* principles, not the lollipops described in *The Saturday Evening Post* of March 16. A teacher has to understand mathematics, elementary and advanced, to know what is basic. Forgetting by the pupil is inevitable unless combatted by regularly spaced drills. This argues for fewer topics, more time, correlated mathematics, and twelfth-grade algebra, though many high school seniors do not elect or drop advanced algebra because they want to graduate *with honors!*

If given two angles, trying to use the three-side theorem to prove two triangles congruent is not a sensible approach. The a's in $a+2/a$ cannot be divided (the word cancel is error-inviting) because multiplication, not the addition indicated in $a+2$, is the inverse of division. Models blindly followed involving usual situations invite dependence. Unusual situations involving increasing difficulty maintain interest and foster independence. Teachers who understand and believe in mathematics should give throughout the twelve grade levels experimental mathematics which afford these desirable practices.

3. *Combat cumulative weaknesses by working for a plan which will not allow the promotion of incompetent pupils.* A pupil barely passes or perhaps is "pulled over" in algebra 1. In algebra 2 he becomes weaker; algebra 3, same story. He goes to college and is failed. Had he failed several years earlier, an economical saving would have been effected for both the pupil who would have found himself and the taxpayer. A progressive educationist views the failure of a child as a violation of the child's personality. The opposing traditionist *opposes* unless the child is his son or unless *he* (the traditionist) is paying school taxes. If the adults can be managed, teachers can tactfully guide young people to take the having to work a longer time to perfect this or that school task more philosophically than they take unavoidable, unprovided-for, everyday reverses. While, generally speaking, pupils do not know less of mathematics than they do of other subjects they pass—the accurate mathematics measuring rods merely "find them out"—cumulative weaknesses, in this subject where each unit is definitely based on the preceding one, cannot be overcome until a satisfactory plan not to promote incompetent pupils is evolved. Freedom to fail slow pupils repeatedly is not a complete solution. Better textbooks, one-fourth credit courses, slow-learning groups, minimum-essential groups, groups which may be shifted, and experimental mathematics are possible factors in a solution. Another significant factor is a teacher who, instead of bemoaning a pupil's weaknesses—a practice which reflects on the teaching profession—will use that time and energy to clarify basic principles for the learner. Naturally, the increasing maturity of the student makes the teach-

ing easier each successive year for each successive teacher. Furthermore, any successful plan will have to be based on the indisputable fact that the upper 10% in school today has a one-to-one correspondence with the 100% in school forty years ago.

4. *Provide informational mathematics and concrete practices in quantitative thinking for pupils who retard those who, if not hampered, can by studying pure mathematics benefit themselves and mankind.* Recognized mathematics and mathematics not recognized as mathematics are on every side. Teachers, not apologetically or selfishly but because an intelligent citizenship wants to understand its surroundings, should offer required courses in informational mathematics. Incidentally citizens who are shown that scientific experiments are at a stand-still until mathematical formulas are derived to point the way will more readily support elective courses in pure mathematics for those who lean in that direction.

With multitudinous phases of mathematics to offer, school people are stupid constantly to try to put square-everyday-mathematics pegs in round-pure-mathematics holes. Potential leaders are allowed to mark time in shallow round holes while teachers laboriously *hammer* at the *corners* of the square-peg members of the class. If the different types could be recognized and properly placed, all of them might benefit themselves and society.

Boys and girls, despite everything to the contrary, *learn.* Mathematicians usually believe in broad, cultural, academic education, but who are the mathematicians to judge the learning of others? Intolerance is neither broad nor cultural. Tom, a member of a slow-learning general mathematics class, cannot subtract $1\frac{7}{8}$ from $3\frac{1}{4}$; however, last week Tom sniffing an across-the-street paint factory, told his instructor so much of making paint that she *respects* even if she, as yet, has not *taught* him to subtract $1\frac{7}{8}$ from $3\frac{1}{4}$. John is a repeater in a plane geometry class. March 20 at 11:46 A. M., when his teacher went to his chair to inspect an assigned scale drawing, John whispered: "I am not drawing. I have figured that it is only forty minutes until spring begins."

Who is to judge?

A TEACHER, to be of maximum service to the community in which he lives, should be recognized as an educated man to whom adult members of the community may turn for consultation in intellectual matters. He should be able to participate in community activities and assume his share of leadership. Certainly he cannot function satisfactorily if he is notably ignorant in what are commonly regarded as fundamentals of general culture. With these facts in mind we advocate a breadth of training for teachers of mathematics which will insure a degree of familiarity with language, literature, fine arts, natural science, and social science, as well as mathematics. "Report on the Training of Teachers of Mathematics," *American Mathematical Monthly*, XLII (May, 1935), p. 272.

The American Way of Thinking About the Value of Mathematics as Revealed by Ten of Our Great Leaders

By Edwin W. Schreiber
Western Illinois State Teachers College, Macomb, Illinois

Benjamin Franklin (1706–1709), American statesman, printer and physicist:

"Mathematical demonstrations are a logic of as much or more use, than that commonly learned at schools, serving to a just formation of the mind, enlarging its capacity, and strengthening it so as to render the same capable of exact reasoning, and discerning truth from falsehood in all occurrences, even subjects not mathematical."

Thomas Jefferson (1743–1826), Third president of the United States (1801–1809):

"Having to conduct my grandson through his course of mathematics, I have resumed the study with great avidity. It was ever my favorite one. We have no theories there, no uncertainties remain on the mind; all is demonstration and satisfaction."

Ralph Waldo Emerson (1803–1882), American essayist and philosopher:

"It is better to teach the child arithmetic and Latin grammar than rhetoric and moral philosophy, because they require exactitude of performance, it is made certain that the lesson is mastered, and that power of performance is worth more than knowledge."

Abraham Lincoln (1809–1865), Sixteenth president of the United States (1861–1865):

Abraham Lincoln's short Autobiography, written in 1860, contains the following:

"He studied and nearly mastered the six books of Euclid since he was a member of Congress."

"He began a course of rigid mental discipline with the intent to improve his faculties, especially his powers of logic and of language. Hence his fondness to Euclid, which he carried with him on the circuit until he could with ease demonstrate all the propositions in the six books; often studying far into the night, with a candle near his pillow, while his fellow-lawyers, half a dozen in a room, filled the air with interminable snoring."

Edgar Allen Poe (1809–1849), an American poet and writer of tales:

"The faculty of resolution is possibly much invigorated by mathematical study and especially by that highest branch of it which, unjustly, merely on account of its retrograde operations, has been called, as if par excellence, analysis."

Henry Ward Beecher (1813–1887), American clergyman:

"Thanks to his friend and teacher Fitzgerald, his mathematical training had given him the entire mastery of La Croix's Algebra, so that he was prepared to demonstrate at random any proposition as chance selected—not only without aid or prompting from the teacher but controversially against the teacher, who would sometimes publicly attack the pupil's method of demonstration, disputing him step by step, when the scholar was expected to know with such positive clearness as to put down and overthrow the teacher. 'You must know not only, but you must know that you know,' was Fitzgerald's maxim; and Henry Ward attributes much of his subsequent habit of steady antagonistic defence of his own opinions to his early mathematical training."

Though prepared for the sophomore class at Amherst, he entered as a Freshman, and had much leisure time. "As he himself remarks, 'I had acquired by the Latin and mathematics, the power of study. I knew how to study, and I turned it upon things I wanted to know.'"

Charles William Eliot (1834–1926), president of Harvard University (1869–1909):

"That the great mass of American children should leave school without ever having touched this subject—except, perhaps, in arithmetic, under the head of mensuration—is a grave public misfortune."

Albert Bushnell Hart (1854–), an American historian:

". . . there are two mathematical subjects—algebra and geometry—in which training is the larger element; . . . the exactness of logical reasoning in geometry makes up for some of the loose habits of thought which children get in other subjects. Geometry, properly taught, is one of the most interesting of subjects, and it may readily be allied with drawing and mensuration."

John Dewey (1859–), an American psychologist and educator:

"Mathematics is said to have, for example, disciplinary value in habituating the pupil to accuracy of statement and closeness of reasoning, it has utilitarian value in giving command of the arts of calculation involved in trade and the arts; culture value in its enlargement of the imagination in dealing with the most general ideas. But clearly mathematics does not accomplish such results, because it is endowed with miraculous potencies called values; it has these values if and when it accomplishes these results, and not otherwise. The statements may help the teacher to a larger vision of the possible results to be affected by instruction in mathematical topics. But unfortunately, the tendency is to treat the statement as indicating they operate or not, and thus give it a rigid justification. If they do not operate, the blame is put, not on the subject as taught, but on the indifference and recalcitrancy of pupils."

Charles Evans Hughes (1862–), an American statesman and jurist, Chief Justice of the United States Supreme Court (1930–):

"As I look back upon my own experience I find that the best lessons of life were the hardest. . . . My mother's insistence upon daily exercises in mental arithmetic has been worth more to me than all the delightful dallyings with intellectual pleasures I have ever had. Life is not a pastime, and democracy is not a holiday excursion."

Interesting—
The geometry of your face,
The algebra of your eyes—with their knowns and unknowns,
The calculus of your personality—with its constants and variables,
The radii of your charm that encircles me,
Your very locus,
The force of your smile, that produced becomes the perpendicular bisector of my heart,
And leaves me entangled in the graph you have plotted around me.—Catherine M. Willimen, 34 Somerset Street, Providence, R. I.

The Use of Applications for Instructional Purposes*

By Edwin G. Olds
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Introduction. As a group, teachers of mathematics are respected and admired for their skill and interest in communicating knowledge of their subject. This universal recognition of their high standards of competence is well deserved. There seems to be no other field in which the experts are more willing to examine their knowledge of subject matter objectively and to bend every effort to remedy any weakness perceived. I doubt whether there is another subject in which the instruction is less open to criticism because of violation of pedagogical principles. Having attained leadership by insistence on thorough knowledge and by intelligent attention to instructional techniques, mathematics teachers can be depended upon to use the same qualities in maintaining that leadership.

Teachers of mathematics, and those cognizant of the nature of their instruction, are more and more convinced that greater attention should be paid to the possibility of the use of applications for strengthening mathematics teaching. The appointment of a committee by the Society for the Promotion of Engineering Education for the purpose of compiling problems illustrating the uses of Engineering Mathematics supports this conviction. The willingness of the National Council of Teachers of Mathematics to devote its 1942 Yearbook[1] to the subject of mathematical applications indicates that the officers and directors of our own organization share this belief. And your own decision to attend this meeting at which the use of applications is the subject under discussion, indicates your personal acceptance of this view. I trust that our program this morning will help all of us to make our teaching more vital by the use of pertinent and significant applications.

In considering the use of applications for instructional purposes there are three questions which we need to answer: 1) Why should we use applications? 2) What applications can we use? 3) How shall we use them? In endeavoring to supply partial answers to these three questions I shall direct my remarks at the level of classification spoken of as "high school," comprising, roughly, grades seven to twelve inclusive.

Why use applications? Teachers agree that applications increase interest. The study of history is enthralling because it reveals stories of people who have lived; science is fascinating because it deals with materials which touch our daily existence; on the other hand mathematics seems to be concerned with operations and abstract principles whose chief value seems to lie in their importance as foundations for more intricate operations and principles of deeper abstraction. One way to relieve the drabness of this aspect of our subject is to show how the principle or theorem under consideration is applied. For example, arithmetic progressions seem to be more vital when we discover[2] the following use in engineering:

* Presented before the Seventh December Meeting of the National Council of Teachers of Mathematics at Baton Rouge, Dec. 31, 1940.

[1] The title will be, "Compendium of Mathematical Applications." The author of the present paper is acting as chairman of the committee responsible for this Yearbook. All teachers are urged to help the committee by contributing appropriate material. Illustrations used in this paper may serve to indicate the type of material desired.

[2] Bernard Lester, *Applied Economics for Engineers*, Wiley, New York (1939), pp. 154–155.

Sometimes the amount of yearly depreciation of a machine is computed by the "sum of expected life periods method." For example, suppose a machine costs \$1,000, has a service life of 5 years, and can be scrapped for \$100. At the beginning of the first year it has an expected service life of 5 years left, at the beginning of the second year it has 4 years left, etc.; therefore it has a "life term" of $5+4+3+2+1$ or 15 years. Then the depreciation will become 5/15 of \$900 for the first year, 4/15 of \$900 for the second year, etc., until the scrap value is reached at the end of 5 years' life.

To see how a mathematical law is applied is to gain a better understanding of its meaning. Many times, in fact, one does not grasp a principle at all until he tries to apply it under specific conditions. In an address to the Society for the Promotion of Engineering Education last October, President Wickenden[3] of Case School of Applied Science made the statement, "The student learns principles best in connection with their applications." He listed the three stages of becoming educated as 1) gaining knowledge, 2) organizing it, 3) applying it. And later he said, "One who has acquired an aptitude for applications can acquire new skills with relative ease."

Even if applications were not required for purposes of clarification, practice in making them still would be necessary. The fact that a principle is known, does not guarantee that it will be applied. Only exceptional students will be able to apply mathematics without definite training. The process of perceiving the fundamental components of a concrete problem, classifying them in the proper theoretical frame, and the specializing the general solution to obtain the desired result, must be taught, step by step, for a diversity of practical situations. This is the best way to make it probable that the average student will apply mathematical verities outside of the classroom.

If there were no other reason for use of applications, their value in educating the student, teacher and community as to the worth of mathematics would be ample justification. It is unnecessary to call your attention to the present crisis in mathematics or to describe its nature. Mathematics seems to be losing ground steadily at a time when the world's need for men educated to solve quantitative problems is increasing rapidly. In the city of Pittburgh, for example, a half-year of arithmetic has been deleted from the seventh grade curriculum and another half-year from the eighth. According to a report[4] by Douglass, 56% of our high school population was studying algebra in 1900, but only 25% in 1935. The corresponding figures for geometry were 27% in 1900, and less than 15% in 1935. This means that the great majority of our students elect less than two years of high school mathematics. It appears that college mathematicians at least, do not approve of this trend. On the recommendation of their War Preparedness Committee,[5] the Council of the American Mathematical Society and the Board of Governors of the Mathematical Association of America, have adopted the following resolution:

"That all competent students in the secondary schools take the maximum amount of mathematics available in their institutions. In the case of many schools, additions to the present curriculum will be necessary in order to furnish an adequate background for the military needs of the country."

But, quite aside from emergency considerations, the American people are ready to pay, and pay generously, for anything which gives promise of being valuable to

[3] William E. Wickenden, "The scientific-technological stem of engineering education," an unpublished address presented before the Allegheny Section of the Society for the Promotion of Engineering Education at Pittsburgh, Oct. 25, 1940.

[4] Harl R. Douglass, "Let's Face the Facts," THE MATHEMATICS TEACHER, Feb. 1937, p. 56.

[5] Report of the War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America at Hanover, *Bulletin of the American Mathematical Society*, vol. 46 (1940), p. 712.

their youth. They show unexpected astuteness, however, in refusing to pay for unfulfilled covenants. Therefore, mathematics need have no fear that any less worthy branch of knowledge will supplant it permanently, no matter how glowing the prospectus for the other field may read.

We may have been too well content, however, with the favored position vested in mathematics by tradition to bother to provide proper appreciation for present-day values. It seems possible, in some teachers, to detect that type of dangerous complacency. On the other hand, many instructors exhibit a surprising lack of conviction as to the importance of mathematical education beyond arithmetic for any students save those who require credits for entrance to our institutions of higher learning. There is no need to remark on the deplorability of this latter condition. The salvation for both groups lies in closer attention to current opportunities to utilize elementary mathematical principles.

It is not enough for you to perceive the myriads of mathematical problems outside the covers of textbooks; the contagion of your enthusiasm must spread to your students and thence to the parents and to the community. If our high-school graduates have learned to *use* a respectable portion of the mathematical knowledge they have labored to *acquire*, there will be little danger that they will permit mathematics to be treated inconsiderately. But this type of education must not be delayed too long, for if the present downward trend continues the time may soon be reached when it will be hard to find many students to animate.

What applications might be used? In answering this question, the average teacher is inclined to consider more advanced mathematics first. It is natural for one to prefer a field in which one has thorough competence. While this source is fruitful, it may provide material which is unattractive and inappropriate for that part of the enrolled student body who have no expectation of careers in which higher mathematics plays a leading role.

If we are able to use applications taken from other school subjects, we shall be contributing to the integration of secondary education and to the promotion of mutual understanding and respect among its teachers. Mathematics suffers many bitter criticisms because students lack facility in applying what they have learned to physics, drafting, and home economics. On the other hand, teachers outside mathematics may presuppose knowledge beyond present curriculum requirements, (for example—how to extract cube root), or may incur student contempt by clumsy or immature mathematical procedures. Some of these difficulties may be avoided, and much benefit can be reaped, through conferences arranged to ferret out applications.

An example[6] of the type of application mentioned above is the following:

> The Oppenheimer formula for the coefficient of nutrition is 100 times the arm girth (taken midway between elbow and shoulder) divided by the chest girth (average between girth expanded and girth deflated). For normal nutrition this coefficient should be at least 30.

A third, and sometimes virgin, field for applications, is the assortment of vocations by the exercise of which men and women earn their living. Those of you who have explored this interesting territory know how often material of significance is discovered. Workers in both trades and professions are glad to have you listen to descriptions of their duties and experiences, and you, being alert to the general utility of mathematics, can repay them with helpful suggestions of mathematical genesis. In the process, you will make friends and will accumulate information with which to build practical problems.

[6] Caldwell, Skinner and Tietz, *Biological Foundation of Education*, Ginn and Co., (1931), p. 340.

For example, many a fine problem can be built on the following information:

In laying concrete the amount of sand, stone and cement for a cubic yard of concrete, depends on the size of the gravel used and the richness of the mix. A 1:3:4 mix means 1 cubic foot (or sack) of cement to 3 cubic feet of sand and 4 cubic feet of stone. The following table gives the amount of each material for some of the common mixes.

Quantities of material for one cubic yard of concrete, (using one inch stone and under):

Mix	Cement (in Bbl.)	Sand (in Cu. Yd.)	Stone (in Cu. Yd.)
1:1:2	2.57	.39	.78
1:2:3	1.70	.52	.77
1:2:4	1.46	.44	.89
1:2½:5	1.19	.46	.91
1:3:5	1.11	.51	.85
1:4:7	.83	.51	.89

The most surprising assortment of people devote part of their leisure time to mathematical puzzles. The great majority of our population play or watch games which involve some use of mathematics. Many enthusiasts work on hobbies which require a degree of mathematical knowledge. The activities of all three groups prove that mathematics plays a part in popular recreation. Therefore, these should be sources from which applications of great psychological value could originate.

The central theme of public education is preparation for citizenship. Perhaps this may be intepreted as meaning preparation for daily living, that is for the activities and pursuits which are common to all. Here is the prime source to be minutely inspected for mathematical applications. If we do not find "significant" uses of mathematics here, we cannot support our present position that mathematics should be studied by all. It is not enough to claim that applications exist; we must cite them. Broad generalities do not win arguments; our briefs must be specific.

An excellent illustration of a realistic problem is the following, kindly submitted by Professor Norman Anning:

A car going s miles per hour can, if the driver is alert, if the brakes are perfect, if the pavement is satisfactory, and if everything else is favourable, be stopped in less than d feet, where $d=s+s^2/20$. Practically, the plus sign separates thinking distance from braking distance. Make a table, and then a graph showing how d changes with s. Find, from the formula, d when $s=40$, 45, 50, 70, 100. From the graph (or by solving a quadratic equation) find s when $d=100$, 200, 300. Show that for a speed of 100 "visibility" must be nearly twice as good as for a speed of 70. What should be the speed of a driver in a fog who cannot see clearly more than sixty feet?

Under ideal conditions d is given more precisely by the formula $200(d+5)=(3s+35)^2$. Show that this gives smaller values for d, and show also that the difference is insignificant for the range of speeds which are legal under ordinary conditions.

Few of us would care to dispute the claim that it requires little mathematics to keep alive. Similar assertions about other school subjects could be justified equally well. One can eat and sleep, work and play, marry and buy automobiles,—one can vote, even—without mathematics, or geography, or grammar, or social studies. But what a narrow bovine existence it must be! How lacking in freedom one is when he must depend on others for help in all situations save the most elemental!

Some time ago a student of mine set out to determine what mathematics an electrician's helper needed to know. He found the assignment very difficult because he confused the issue time and again with that of determining what mathematics it would be advantageous for the helper to know. He kept insisting that the helper needed to know so and so, if he wanted to become a good electrician, if he wanted to have his own business, if he didn't want to be just a helper all of his life. There is no necessity to emphasize the point further. In daily living, just as in work as an electrician's helper, very few problems have to be solved, but we cannot hope to live the fuller life if we are content with the irreducible minimum of knowledge.

Therefore when we search life for applications, let us keep in mind that we are looking, not only for the problems that the average man solves, but also for those which he should solve. And notice that his failure with many problems is due as much to lack of facility in the use of knowledge as it is to the lack of the knowledge itself.

How to use applications? If you agree that applications should be used, and decide what types are available, then you must plan how to use them. At once you perceive several difficulties and, in trying to surmount them, you will begin to realize why progress in this direction has been painfully slow.

The first difficulty is lack of time. From grade seven to the last year of graduate work, every mathematics teacher has his courses simply packed with principles which in his judgment the student needs desperately. To find time for worthwhile achievement in teaching applications, we must delete some of these principles, precious though they may seem. To make such a reduction may appear unfortunate, but ripe judgment fosters the conviction that the amount of ground covered is not nearly so important as the depth to which the ground is cultivated. If applications add depth, they are worth the loss in area entailed.

Passing next to the perplexing issue of materials to be used one is struck by the stupendous task confronting the individual teacher of compiling a representative file of applications. After teaching six or seven classes of present-day magnitude, even the most conscientious have little energy left for painstaking investigation of original sources. Still, some teachers have been able to collect, bit by bit, surprisingly large amounts of interesting information. As you know, applications are scattered hither and yon. A text in Biology may yield two or three; an advertisement may suggest another; the teacher of cooking may offer a few; and occasionally an evening of bridge may contribute something. However, no one of these may be appropriate to illustrate the next day's principle, and if the reference library is inadequate, that particular theory may go unadorned and unpointed.

Many teachers have hesitated to use applications because their students lacked the background necessary to appreciate them. When choosing applications it is obvious that we must be guided by student interests and environment. The fields of application must intersect, or at least be tangent to, student consciousness, even though it does not seem obligatory, or even desirable, that the overlapping be complete. Why should there be such a taboo on teaching other things beside mathematics in a mathematics class? Do we not constitute a force organized to educate our youth and, therefore, are we not responsible for contributing in every possible way to that education? If a student lacks the underlying facts necessary to make an application significant, why not provide them? Why stifle procedures which lead to integration, especially when their use promotes mathematical education?

Not long ago the author received an advertisement stating:

"On a cash loan of $150.00, complete monthly payments on our 18-month plan will be only $9.57."

Is it a defensible procedure to take time in the mathematics period to give students the education in financial methods necessary to intelligently appraise the above statement?

In rare instances, it seems to be the teacher's background, rather than that of the student which causes the difficulty. Teachers are specialists and, like most specialists, feel a certain lack of confidence when they sail too far from their home port. The United States, as a whole, is becoming a nation whose workers concentrate on the swifter and more accurate performance of duties of less and less individual scope. From the time that an automobile spring starts as strips of steel until it emerges as a finished entity at the loading dock, thirteen different workmen have a part in its production.[7] Each must apply to his task such a high degree of concentration that he misses the opportunity to study the process as a whole. In fact, it may be only when something goes wrong, that he perceives the interdependence of his work with that of his associates. May the time never come when our work with human materials is subdivided so finely, but, if it should come, let us hope that most of the mathematics instruction will be given by teachers who have not been required to sacrifice broad intellectual experience to the demand for intense specialization.

Conclusion. In the course of this paper, it has been stated that applications should be used in order to increase interest, to aid understanding, to provide practice in applying knowledge, and to give the students and the community an appreciation of the value of mathematical education. It has been pointed out that applications may be found in higher mathematics, in other school subjects, in vocations and avocations, and in the activities of our daily life. Suggestions have been offered in regard to the difficulties of finding time and material, and the matter of lack of background has been discussed.

In conclusion the firm conviction must be expressed that greater attention should be paid to the use of applications for strengthening mathematics teaching. As teachers, you hold the highest rank for your intelligent and efficient performance, but your results will become more vital and more generally appreciated if you will make mathematics richer by an increased use of applications for instructional purposes.

[7] Henry Ford, "Mass production," *Encyclopedia Britannica*, (14th ed.) vol. 15, p. 39.

Snowflakes

A trillion snowflakes fall
And each hexagonal,
Not one, not even one
Escapes that magic spell.

A trillion snowflakes fall
And each one different;
Varied their tracery,
The rule is adamant.

I saw them yesterday
And wondered, "Is it true
That everything in life
Follows this same rule?"

Is each soul patterned too
And in its separate ways
With swift and sure design
Its pattern still obeys?

I saw the snowflakes fall
Each separate in design.
Are other traceries
Still different from mine?

I asked of fate and God,
"Can man be bound and free,
Is there a unity,
Is no one just like me?"

My reason says it's true.
In gentle snowflake's fall,
In sparrow's broken wing,
God ruleth over all.

—Benjamin Rood Larrabee

THE ART OF TAEACHING

By Marion G. Eckel
Kelly High School, Chicago, Illinois

In the November 1940 issue of The Mathematics Teacher Mr. Katra's article on "Teaching the Subtraction of Signed Numbers" describes the use of a device which may be very easily extended to the teaching of multiplication and division of signed numbers.

One point about subtraction, not mentioned by Mr. Katra, is the logical transfer to "changing the signs" of terms inclosed in parentheses preceded by a minus sign when the parentheses are removed. Pupils have become accustomed to thinking of subtraction as opposite in meaning to addition and consequently make fewer errors in "mechanical operations."

The use of the device in the other fundamental operations is as follows:

Multiplication

The sign of the multiplier is taken as a *sign of operation.*

In $(+3)\times(+4)$, the $(+3)$ is to be used four times as an addend; by placing the marker on (0) and moving upward, the direction indicated by the sign of the multiplicand, the result is $(+12)$.

$(-3)\times(+4)$, the (-3) is to be used four times as an addend, the marker proceeds downward to (-12).

$(+3)\times(-4)$, the $(+3)$ is to be used four times as a subtrahend, therefore the marker proceeds in the opposite direction from the direction indicated in the sign of the multiplicand, or downward, and rests on (-12).

$(-3)\times(-4)$, the (-3) is to be used four times a subtrahend, therefore the marker proceeds in the opposite direction from that indicated by the sign of the multiplicand, or upward, and rests on $(+12)$.

Division

$(+12)\div(+4)$; place the marker on (0), to reach $(+12)$, the marker would have to proceed upward, the direction indicated by the sign of the divisor, therefore the $(+4)$ must have been used as an addend three times and the result is $(+3)$.

$(+12)\div(-4)$; place the marker on (0), to set the marker on $(+12)$ it must move in a direction opposite to that indicated by the sign of the divisor, therefore was subtracted and the result is (-3).

$(-12)\div(+4)$; place the marker on (0), to reach (-12), the marker must proceed downward or in the opposite direction from that indicated by the sign of the divisor, therefore was subtracted and the result is (-3).

$(-12)\div(-4)$; place the marker on (0), to reach (-12), the marker must proceed downward or in the direction indicated by the sign of the divisor, therefore was added and the result is $(+3)$.

By using the scaled line device in this manner there seems to be little confusion. The methods are consistent and logical and pupils are able to formulate their own rules very quickly.

Division may be taught deductively following multiplication; however I have found that teaching the four operations on the scale brought the best results.

EDITORIAL

What Is the Next Move for the National Council?

ACCORDING to *The Bergen* (New Jersey) *Evening Record* of Saturday, November 16, 1940, Dr. William H. Kilpatrick, said that he would eliminate mathematics from the regular curriculum and bury Latin and Greek.

"They don't have any value in training the brain; in fact mathematics tends to have an opposite effect," he said.

"Only by studying current controversial questions will students learn to think," he said. "Teachers shouldn't present them with the answers. They won't learn to think by studying dead controversies," he said.

This is typical of the kind of criticism mathematics is receiving from many educationists all over the country, and many of them, like Professor Kilpatrick, offer the remedy he suggests, of teaching controversial questions instead. The trouble with such suggested remedies is that they can guarantee no better thinking among secondary students than does the present teaching of mathematics, bad as it may be in some places. The point is that controversial questions can be stupidly taught, and are so taught in places where there are those who have believed that Professor Kilpatrick and others like him are right. Merely to throw out a subject because it is not generally well taught or learned, and to put in its place some untried subject which may also be faulty, will not solve the problem and our critics ought to know that.

The National Council of Teachers of Mathematics knows that the mathematics in the schools should be reorganized for teaching purposes, and it has spent a great deal of time and energy in preparing articles, yearbooks, and reports to help ameliorate the situation. This is all to the good. Its next move is to perfect some plan to offset much of the unfair and unsound criticism that is being leveled at the teaching of mathematics, and then to see to it that something is done to guarantee that the recommendations of the recent reports of national committees on mathematics for improvement may have a chance of being adopted in the schools.

The blame for the present situation in mathematics should not be placed upon the teachers of the subject, although they may have some responsibility. The fault lies with our entire educational philosophy or lack of it, and the way our educational system is administered. Does anyone think that the teaching of mathematics is any worse than the teaching of other subjects? For example, are poets, novelists, and the like being turned out in larger numbers today than formerly by the English teachers? Are the Plutarchs, the Gibbons, the Bancrofts and the Beards being trained in larger numbers by the teachers of history, or are these teachers really producing by their instruction a higher type of citizen? And so we might go on through all the great fields of knowledge. No, the answer is that the trouble lies deeper. By and large, the teachers are doing about as well as can be expected under the circumstances, but clearly the best is none too good!

It is to be hoped that the Report of the Joint Commission and the report of the committee on "The Function of Mathematics in General Education" for the "Commission on Secondary School Curriculum" of the Progressive Education Association will be of great service in reorganizing and improving the curriculum, but we know that will not be enough. Unless the work of the Commission and the Committee are carried on continuously by the teachers of mathematics, much of their work will have been in vain. After all, the mere fact that reports are made settles nothing one way or the other. If teachers do not read the reports and then do something, little progress will be made.

W. D. R.

IN OTHER PERIODICALS

By NATHAN LAZAR
The Bronx High School of Science, New York City

The American Mathematical Monthly

May, 1940, Vol. 47, No. 5

1. Report of the Committee on Tests, pp. 290–301.
2. Mulcrone, T. F., "On the Equations of Conical Surfaces, pp. 302–303.
3. Mathematics Clubs, pp. 312–317.
   a. Suggestions for Presenting a Topic before the Mathematics Club.
   b. Readings in Mathematics for Summer Leisure.
   c. Club Reports, 1938–39.
   d. Recreations.

Association of Mathematics Teachers of New Jersey

Yearbook, 1940

1. Starke, Emory P., "A General Survey of Mathematics in the Social Studies," pp. 5–11.
2. Rolfe, Stanley H., "An Evaluation of the Elementary School Program in Arithmetic," pp. 12–18.
3. Jahn, Lora D., "Meeting the Challenge of Youth from the Point of View of the Curriculum of the Junior High School," pp. 18–24.
4. Schlosser, Jerome, "Meeting the Challenge of Youth through Mathematics Club Programs, Bulletins, and Procedures," pp. 24–29.
5. Hildebrandt, E. H. C., "Meeting the Challenge of Youth through Extra-classroom Activities," pp. 30–34.
6. Fehr, Howard F., "Mathematical Modes and Processes Applicable to Social Studies," pp. 35–41.
7. McMackin, Frank J., "The High School Is Meeting the Challenge of Youth," pp. 42–47.

Bulletin of the Kansas Association of Teachers of Mathematics

October, 1940, Vol. 15, No. 1.

1. "Report on Kansas Mathematics Test Number One," prepared by the Placement Test Committee, Kansas Section of the Mathematical Association of America, pp. 3–17.
2. "Why Mathematics?—A Symposium," pp. 17–20.
   a. Nock, S. A., "Dangerous Thoughts."
   b. Grimes, W. E., "Mathematics and Living."
   c. Howe, Harold, "Mathematics in Economics."
   d. Pittman, Martha S., "Mathematics in Foods and Nutrition."
   e. Bayfeld, E. G., "Mathematics in Milling."
   f. Swanson, C. O., "Mathematics in Milling."
   g. King, H. H., "Mathematics in Chemistry."
   h. Laude, H. H., "The Value of Mathematics in the Study of Agronomy."
   i. Ibsen, Herman L., "Mathematics in Genetics."
   j. Farrell, F. D., "Truths about Education."

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1. Finkel, Benjamin F., "A History of American Mathematical Journals," (continued), pp. 27–34.
2. O'Toole, A. L., "Insight or Trick Methods?" pp. 35–38.

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December, 1940, Vol. 40, No. 9.

1. Yates, Robert C., "Addition by Dissection," pp. 801–807.
2. Johnson, J. T., "New Number Systems vs the Decimal System," pp. 828–834.
3. Corliss, John J., "Formulas for Volume by Simple Algebra," pp. 846–850.
4. Franzen, Carl G. F., "The Mathematics of The Modern Curriculum," pp. 862–866.
5. Hitt, James K., "A Note on Simple Interest," pp. 873–875.
6. Read, Cecil B., "Inconsistencies in Number Classification," pp. 876–877.
7. Schreiber, Edwin W., "A Current Mathematical Bibliography for Teachers in Service," pp. 881–883.

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December, 1939, Vol. 6, No. 4.

1. Franklin, Philip, "The Four Color Problem," (conclusion), pp. 197–210.
2. à Kempis, Sister Mary Thomas, "The Walking Polyglot," (Maria Gaetana Agnesa), pp. 211–217.
3. Curiosa, pp. 218.

a. G., "Numbers with Reversed Order of Digits."
b. Juzuk, Dov, "An Interesting Observation on the Series of Natural Numbers."

4. Bradley, A. Day, "The Mathematical Notebooks of James Boone, Jr.," pp. 219–227.
5. Recreational Mathematics.
   a. Shulman, David, "The Lewis Carroll Problem," pp. 238–240.
   b. Coxeter, H. S. M., "The Regular Sponges, or Skew Polyhedra," pp. 240–244.
6. Curiosa, p. 246.
   a. Whitlock, W. P., Jr., "A Family of Giant Pythagorean Triangles."
   b. Pollack, Sigfried, "Π as a Mystic Number."

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2. Clement, S. L., "Geometry Comes to Life," *Clearing House*, 15: 92–95, October, 1940.
3. Dahle, C. O., "Verbal Thought and Overt Behaviour of Children during their Learning of Long Division," (Digest of a Dissertation), *Journal of Experimental Education*, 9: 1–8, September, 1940.
4. "Improvement of Classroom Teaching; Business Arithmetic," *Twelfth Yearbook of the Eastern Commercial Teachers Association*, pp. 83–93.
5. Leonhardy, A., "Secondary School Mathematics and the Students' Needs," *Secondary Education*, 9: 229–233, October, 1940.
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7. Shelton, H. S., "Wanted—a New Mathematical Syllabus," *Journal of Education* (London), 72: 450–451, October, 1940.
8. Strangways, D. H., "Organization in Middle School Geometry," *School* (Secondary Edition), 29: 129–132, October, 1940.
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NEWS NOTES

Mathematics Teachers Organize New Section

Many mathematics teachers in New Jersey have long felt the need for a specialized section of the Association of Mathematics Teachers of New Jersey devoted to higher mathematics, applied mathematics and teacher training.

The first move to organize such a section was taken at the New Jersey College for Women on Thursday, November 28, at 7:30 P.M. The new section was named Section 4 and is closely affiliated with the state organization of mathematics teachers. The following officers were elected:

Professor Albert E. Meder, Jr., New Jersey College for Women, Chairman
Ferdinand Kertes, Perth Amboy High School, Organizer
Dr. E. H. C. Hildebrandt, Montclair State Teachers College, member of Council
Donald N. Armstrong, Roselle Park High School, member of Council

After the election of officers, the assembled group was addressed by Robert M. Walker, Assistant Professor of Mathematics, New Jersey College for Women. The topic of his address was "Mathematics as Used in Life Insurance." A half hour of lively discussion from the floor followed the address.

Mr. Walker prefaced his remarks by outlining the Actuarial Society's requirements for entry into the actuarial field. He continued by describing the organization and functioning of a life insurance company. In the course of his address he made continuous references to this set-up in order to explain, in non-technical language, how mathematics is used in the calculation of premiums, reserves, cash surrender values and dividends.

The Women's Mathematics Club of Chicago and Vicinity will hold its next meeting December 7 in Mandel Brothers' two "Ivory Rooms" on the ninth floor, Wabash Avenue side at 11:50 A.M.

Dr. W. C. Krathwohl of the Illinois Institute of Technology will talk on "What the Armour Cooperative Plan Really Is."

To open the meeting Mrs. Bernice Engels of Gary and Mrs. Stacia Springer of Crance High School will discuss briefly current problems of teachers of mathematics.

The hostesses for the meeting are Ruth Ketler of Fenger high school, Margaret Collins of Lane, Janette Spaulding of Bloom Township and Norma Sleight of New Trier.

Lenore H. King, Publicity Chairman

The November meeting of the William Wallis Mathematics Club of Washington, D. C. was held at Western High School of that city. After a very enjoyable tea, at which the mathematics faculty of the school acted as hostesses, the members of the Club were privileged to hear a very interesting and worth-while talk on "The Teaching of Mathematics" by Mr. William W. Hart, until recently the Associate Professor of Mathematics of the School of Education, University of Wisconsin. Miss Veryl Schult, Supervisor of Mathematics of Washington, D. C., introduced the speaker and welcomed two guests from a neighboring city, the supervisors of mathematics of Baltimore, Maryland.

The Mathematics Section of the High School Conference (the Illinois Association of the National Council of Teachers of Mathematics) met on November 1, 1940 in 112 Gregory Hall on the University of Illinois campus, Urbana, Illinois. The theme of the program was: "Relational Thinking in Action."

The chairman, Mr. H. G. Ayre of Western Illinois State Teachers' College, Macomb, called the meeting to order and read a greeting to the section from Dr. W. D. Reeve of Columbia University in which he said, in part: "Mathematics is in a peculiarly fortunate position at the present time in that the defense program is bound to accelerate interest in our subject. Boys who formerly took little if any interest in Mathematics are now discovering that they are not able to get into some branches of the service unless they have had a course in trigonometry.

"It seems to me that we mathematics teachers have a wonderful opportunity to put our subject where it rightfully belongs in the curriculum, first, by teaching it better, and second, by making sure that people are made aware of its importance among the great branches of learning."

The first speaker of the morning session was Miss Mildred Taylor of Fenger High School, Chicago, who spoke on "Doing Relational Thinking in High School Algebra," showing that it must be introduced in algebra because there we are paving the way for all the advanced Mathematics courses of both high school and college. She pointed out the necessity of giving an enriched course for the college preparatory student and at the same time a course full of practical, concrete problems for other students. Many of these examples she illustrated with drawings and models.

Mr. Joseph A. Nyberg of Hyde Park High School, Chicago, next spoke on, "Doing Relational Thinking in High School Geometry," pointing out the ever increasing necessity for teaching pupils to think as straight as possible by omitting some of the formal deductive geometry and teaching logic in order that they may better cope with life situations. Mr. Nyberg advised every teacher to read the two reports, namely, the one by the Joint Commission of the Mathematical Association of America and the National Council, and that of the Progressive Education Association.

After the meeting had recessed for three minutes, Mr. A. E. Katra, made the announcement encouraging membership in the National Council of Mathematics Teachers and to secure buyers for the fifteenth yearbook.

Dr. John R. Mayor, of Southern Illinois State Normal University, Carbondale, our third speaker, addressed us on "Relational Thinking as a Criterion for Success in College Mathematics," demonstrating its use in the unified course in college mathematics which includes college algebra, trigonometry, analytical geometry and elementary calculus. He also used this quotation, "Mathematicians do not study objects, but relations between them."

The chairman appointed the nominating committee as follows: Mr. W. Barczewski, Waukegan Township High School, Chairman, Miss Alvena Bamberger, and Miss Lucy Glasscock. He also announced an executive committee meeting following the morning session.

Then Dr. W. C. Krathwohl of Illinois Institute of Technology, Chicago, gave us a stimulating talk on "Relational Thinking Through Visual Aids, Clubs and Exhibits." To illustrate the point that students lack the ability to visualize, he told of the visualization test given to incoming students in which they are asked to relate models, with the same size and shape, whose sides are marked differently.

Under the heading of "Visual Aids" he mentioned the use of models, charts, motion pictures, colored chalks, and giving of specific directions for seeing better. The best use of models as visual aids is to have the student make one, making his own drawing first thus using relational thinking in the transfer from drawing to model. If this cannot be done, let him handle models which have been made by others. He also demonstrated the ease of model-making with corrugated cardboard, sharpened steel wires and radio clips.

On the subject of charts he said, "Show every detail first which it to go in it. Then have the student make it no matter how rough."

"Motion pictures in mathematics, to accomplish the most, should be of the animated cartoon type. There are comparatively few of these yet," he continued.

"If your students sit before you as though they had packed their brains in cold storage or had a bad cold in the head, it is too much white chalk, teacher. Use colored chalks and see them brighten up."

"It takes only two to make a club. The best club I ever was in was one of that sort."

In clubs where the students pick the topics to discuss, they are sure to appeal to their interests.

Miss Henrietta Terry, of the University High School, Urbana, as chairman for the Mathematics Exhibit made a brief announcement thanking everyone who had contributed and inviting all to see it in Room 13 of Gregory Hall.

The meeting was then adjounred until 2:00 P.M.

The third annual Mathematics Luncheon was served in Latzer Hall of the University Y.M.C.A. There were about 115 present and approximately 50 could not be accommodated.

Mr. Ayre called upon Dr. Miles C. Hartley, chairman for the luncheon arrangements, who introduced Dr. Harold W. Bailey, formerly of the mathematics department, now head of the personnel bureau of the University, as the guest speaker. Dr. Bailey's subject was, "The Relation of the Mathematics Teacher to the Guidance Program." He pointed out to us the need of better guidance in high school so that students may come to the University with the requirements necessary for the courses they wish to pursue. He also illustrated with cases of students he has interviewed, the necessity for aiding students in discovering their real aptitudes so that they are not wasting time and money after they get into college by waiting to learn there that they are entirely unfitted for the things they are trying to do.

The afternoon session was called to order by Mr. Ayre at 2:10 P.M. He opened the meeting by reading the greeting from Miss Mary A. Potter, President of the National Council of Mathematics Teachers. She called to our attention the Christmas meeting to be held in Baton Rouge, Louisiana, from December 30 to January 1. In mentioning the meeting to be held at the

Chelsea Hotel, Atlantic City, February 21 and 22 she wrote: "Since it seems that failures in economics and social science lead us into wars which have to be won by the physical sciences reinforced by Mathematics, the theme of 'Mathematics in a Defense Program' was chosen for this meeting. We hope to hear of the progress that has already been made by the War Preparedness Committee of the Mathematicians. The Visual Aids Committee expects to show us some New English Mathematics films. Lancelot Hogben of *Mathematics for the Million* fame has consented to be our speaker."

Anyone having material on the subject "Applications of Mathematics" for the seventeenth yearbook is asked to send it to Mr. E. G. Olds, Carnegie Institute, Pittsburgh, Pennsylvania.

The minutes of Illinois Section of the National Council for 1939 were read and approved.

Mr. Barczewski gave the following report of the nominating committee: Miss Anice Seybold of Monticello for Chairman, Miss Frances Innes of Dundee for vice-chairman, and Mr. Walter Willis of Dupo for secretary. Since there were no further nominations, a motion was made and seconded that the secretary cast a unanimous ballot.

Dr. J. T. Johnson of Chicago Teachers' College, then spoke on the subject, "The Principle of Relational Thinking in the Teaching of Arithmetic." Concerning the problems of the transfer of training there is no agreement as to the amount of transfer, nor is there an adequate explanation for its cause.

But two theories of Thorndike and Judd remain valid, namely, that of identical elements and that of teaching by generalization. Transfer occurs where similar elements are recognized; or "We learn by experience." Also the higher the intelligence, the more transfer takes place. Teaching for transfer in arithmetic is relational teaching.

The teacher should warn against negative transfer by giving the correct form in comparison with the incorrect; for example, such as $6\times 0=0$, not $6+0=0$, or if $x^2=4$, by taking the square root of each member; but $4/9$ does not $=2/3$ by taking the square root or the numerator and of the denominator.

Mr. Ayre then introduced Dr. F. L. Wren of George Peabody College for Teachers, Nashville, Tennessee, who addressed us on the subject, "The Role of Relational Thinking in Teaching Mathematics." He divided his topic into three subheadings: (1) What do we mean by relational thinking? (2) What are the characteristics of it? (3) What is the role of relational thinking in teaching? Thinking he defined as, "that process by which new experiences are oriented in view of the past and present."

"Relational thinking is that process through which concentrated effort is made to isolate any technique, problem or concept so that generalization will take place leading to abstraction.

The remainder of the afternoon session was devoted to a Question Box with Dr. Mayor, as Master of Ceremonies, and the speakers of the day and Miss Anice Seybold, representing the National Council for Miss Henrietta Terry, as the experts.

On October 26, the Kentucky Council of Mathematics Teachers and the Kentucky Section of the Mathematical Association of America held their first joint meeting at the University of Kentucky. Approximately eighty teachers attended. The following program was given:

"The Training of High School Mathematics Teachers," E. D. Jenkins, Eastern State Teachers College.

"The Place of Mathematics in Secondary Education,"
- a) From the junior high school view point, Russel, Garth, Louisville Junior High School.
- b) From the senior high school view point, Miss Mary E. Clark, Henry Clay High School, Lexington.

"Mathematical Models," W. L. Moore, University of Louisville.

"High School Preparation for College Mathematics," W. R. Hutcherson, Berea College.

"A Pursuit Problem," H. H. Downing, University of Kentucky.

"State Requirement, in Mathematics for Students and Teachers," Mark Godman, of the State Department of Education.

After the program a luncheon was held for the group in the Student Union Building on the campus. Before adjourning, the group decided to continue to have a joint meeting annually, and decided to hold it in the fall at the same time of the meeting of the Kentucky Association of College and Secondary Schools.

At the meeting of the Mathematics Section of the Arkansas Education Association, November 8, the following officers were elected for the coming year:

President: Minot B. Dodson, Jonesboro, Arkansas.

Secretary: Miss Louise Donaldson, Senior High School, North Little Rock, Arkansas.

The program of the meeting was a talk by F. L. Wren on "The Role of Relational Thinking in the Teaching of Secondary Mathematics." About 125 teachers were at the meeting.

Davis P. Richardson

The second meeting of the Men's Mathematics Club of Chicago and the Metropolitan Area was held on Friday, November 15, 1940. Mr. Justin Nicolet spoke on the topic "Mathematics—The Subway Designer's Tool."

Following Mr. Nicolet's talk a new sound-film of the subway was shown. It is entitled "Streamlining Chicago."

The Fifteenth Annual Conference of Teachers of Mathematics was held at Iowa City, Iowa, on October 11 and 12, 1940. The following program was rendered:

FRIDAY MORNING, OCTOBER 11, 1940
North Room, Old Capitol
H. L. RIETZ, *presiding*

10:00 A.M. Address: Creative Imagination through High School Mathematics. Gertrude Hendrix, Eastern Illinois State Teachers College, Charleston.

10:30 A.M. Address: Developing Understanding in Junior High School Mathematics, Katherine Young, Wallace Junior High School, Waterloo.

11:00 A.M. Address: The Definite Integral, Lewis E. Ward, University of Iowa.
Discussion.

FRIDAY AFTERNOON, OCTOBER 11, 1940
North Room, Old Capitol
E. N. OBERG, *presiding*

1:30 P.M. Address: The Arithmetical Responsibility of the Secondary School, F. L. Wren, George Peabody Teachers College, Nashville.

2:00 P.M. Address: The Mathematics Needed for Certain Scientific Courses, George H. Nickle, Keokuk High School, Keokuk.

2:30 P.M. Address: Curriculum Trends and the Implications for High School Mathematics, George B. Smith, University of Iowa.
Discussion.

FRIDAY EVENING, OCTOBER 11, 1940
Iowa Memorial Union
River Room

6:00 P.M. Conference Dinner
Ruth Lane, presiding
Play: A Case of Figures
University High School Pupils.

SATURDAY MORNING, OCTOBER 12, 1940
North Room, Old Capitol
L. A. KNOWLER, *presiding*

9:30 A.M. Address: The Concept of Freedom through High School Mathematics, Gertrude Hendrix.

10:00 A.M. Address: The Place of Mathematics in a Functional Curriculum, F. L. Wren.
Discussion.

The seventieth regular meeting of the Association of Mathematics Teachers of New Jersey was held at the Hotel Traymore in Atlantic City, N. J., on November 9, 1940. President, Mary C. Rogers of Westfield presided. The following program was given:

THE PLACE OF MATHEMATICS IN
EDUCATION FOR SOCIAL CHANGE
Dr. Ernst R. Breslich
Department of Education
University of Chicago

New Jersey's Response:

1. Number—The Basic Language for Expressing Quantitative Relations, Miss Anne Graffam, Vice-President, New Jersey Elementary Mathematics Association.
2. Mathematics Which Functions in the Expanding Social Life of the Early Adolescent, Chester A. Olinger, Junior High School, Collingswood, N. J.
3. Socio-Economic Mathematics for Senior High School Students, Chauncey W. Oakley, Senior High School, Manasquan, N. J.
4. Mathematics and Technology, Dr. Edward Baker, Associate Professor in Mathematics, Newark College of Engineering.

HAPPENINGS OF THE ASSOCIATION

South Jersey Meeting: The Southern Section of the Association of Mathematics Teachers of New Jersey met at the State Teachers College, Glassboro, New Jersey, on Tuesday, October 16, 1940. The theme for discussion was "Adventuring in Geometry."

The afternoon session, called at four forty-five, was given over to demonstration lessons on the Junior High School and Senior High School levels. Both classes were from the Glassboro Public Schools.

The Senior High School class was conducted by Dr. C. N. Stokes of Temple University. He showed how a beginning class in Geometry can weigh statements from daily life and draw conclusions. From this experience he guided the class in building up a proof from simple geometric statements.

Demonstrating with a group of eight grade pupils, Mr. Paul J. Whiteley of the Cheltenham Junior High School, developed the concept of volume with particular reference to the formula for the volume of a rectangular solid. This work was made extremely meaningful by the skillful correlation of concrete experiences with abstract reasoning.

The demonstration lessons were followed by dinner at the College. After dinner, visitors were given an opportunity to see an exhibit of work prepared by the State College students.

Promptly at seven o'clock the evening session was called with Dr. Stokes as the speaker of the evening. Emphasizing teacher responsibility for the training of thinking individuals, Dr. Stokes explained how skillful direction in the making of mathematical analyses in Geometry followed by careful guidance in forming the generalizations resulting from these discovered relationships, can develop a way of thinking.

The meeting was in charge of: Miss Marion Lukens, Camden High School, Chairman; Miss Kjersten Nielsen, State Teachers College, Glassboro; and Mr. Chester A. Olinger, Junior High School, Collingswood.

N.B. The South Jersey programs have been so well attended, and have created such widespread interest, that your President, acting upon the advice of Miss Lukens, has doubled the committee personnel to provide for expansion of activities. The newly appointed committee members are: Mr. Carl E. Heilman, Senior High School, Paulsboro; Mrs. Jessie M. Lewis, Salem High School; Mr. Dwight L. M. Powell, Senior High School, Cape May Court House.

BOOK REVIEWS

Development of the Minkowski Geometry of Numbers. By Harris Hancock. The Macmillan Company, New York, 1939. xxiv+839 pp. Price, $12.00.

The Geometry of Numbers is a name for a branch of the Theory of Numbers. It is usually associated with the name of Herman Minkowski (1864–1906) whose book the *Geometrie der Zahlen* ranks as one of the most outstanding works in the field of the Theory of Numbers. Minkowski, however, was not the first who created the link which connects geometry and numbers. Dirichlet and Hermite made use of the geometric approach in the study of problems of the Theory of Numbers. But the name of Minkowski is generally associated with this field because he developed it systematically. Moreover, his work opened new roads to research and stimulated others, especially Hilbert whose work in the direction of the geometrization of the Analysis resulted in such new ideas as the Hilbert Space.

The field of Geometry of Numbers does not belong to elementary, or even undergraduate mathematics. This is unfortunate, because there are many elementary ideas in the Geometry of Numbers which should be a part of a mathematics course which prospective teachers of mathematcs should have while in training.

Dr. Hancock presents a book which fills a gap in the mathematics literature. For this contribution he deserves the gratitude of those who are interested in mathematics. Minkowski's *Geometrie der Zahlen* is a book which can be mastered by few, and in places its reading is a Herculean task for many. Dr. Hancock set out to develop Minkowski's work in this field and to make the Geometry of Numbers comprehensible. This he has done admirably well.

It is impossible to enumerate all the topics which the Geometry of Numbers covers. The development of this subject, as presented by Dr. Hancock, is complete, and for a person with mathematical maturity the book should be not only fascinating, but comparatively easy to read.

Some of the topics in the Geometry of Numbers may be applied to the study of Plane and Solid Geometry. Dr. Hancock's book was not written with this aim. But the material presented by him, and while developing it he drew not only upon Minkowski's *Geometrie der Zahlen* but on his other works as well as on his personal correspondence, contains sufficient hints which could be successfully applied, even on an elementary plane, to the study of some geometric properties of two and three dimensional figures. The following topics may be mentioned because they might be of interest to those who prefer to study geometry and who desire to draw on them in order to develop some interesting properties of geometric figures and design. The extremal properties of convex geometric figures as well as the problem of isoperimetry may be treated from the point of view of Geometry of Numbers. The study of the extremal properties was made by Minkowski (*Volumen und Oberfläche.* Math. Annalen, Bd. 57, pp. 447–495). His theorem, given by Dr. Hancock in Art. 5, *A convex body having a body* $=8=2^3$ *and having a lattice point as its center always contains besides the center at least one other lattice point either within or on the boundary*, if restated in a more elementary form: *If the area of a centrally symmetric convex figure M, whose center of symmetry is a lattice point, is* $=4$, *then M covers besides its center some other lattice points*, may be made the basis of the study of the geometry of the repeating design as well as of the problem of filling the plane with polygons. The problem of symmetry on a sphere as well as the problem of filling the space with polyhedra may be based on the theorem as stated by Dr. Hancock.

The study of the isoperimetric properties of convex geometric figures (plane and solid) may be based on the above theorems. Although this problem may be attacked from many points of view, as, for example, synthetically (Jacob Steiner, *Systematische Entwickelung der Abhängigkeit geometrischer Gestalten voneinander, 1832*), or analytically (R. Sturm, *Maxima und Minima in der elementaren Geometrie*, Berlin 1910), Minkowski's approach is nevertheless very elegant and not difficult.

Perhaps someone will find time and devote himself to the elementary development of the Geometry of Numbers so that the teachers of secondary mathematics may learn some of its phases. This problematic author will thus make a real contribution to the training of mathematics teachers in this country.

AARON BAKST

Descriptive Geometry. By James T. Larkins, Jr. Prentice-Hall, Inc., 1939. viii+314 pp. Price, $2.50.

This text was designed by the author to be understandable, interesting, and broadly practical. It is the outgrowth of many years of studying and analyzing the difficulties that students experience with descriptive geometry. The subject is presented informally so as to accelerate the students' interest.

The book is intended for students in all branches of engineering and other scientific courses. The author does not attempt to cover the entire field of descriptive geometry. Only those things that develop the student's judgment and visualizing ability are included.

The book is divided into four sections as follows:

1. The fundamental principles of points, lines, and planes.
2. The piercing point as related to lines and planes.
3. Revolution and counter-revolution.
4. Application of Sections 1, 2, and 3 as related to plane problems and single- and double-curved surfaces of revolution.

W. D. R.

Teaching Mathematics in the Secondary Schools. By J. H. Minnick. Prentice-Hall, Inc., 1939. xiv+336 pp. Price, $3.00.

Dean Minnick has had an unusual opportunity to observe the many changes that have been made in the teaching of mathematics in recent years, and he is well qualified to pass judgment as to which changes are significant for the teachers of mathematics in this country. The book is an outgrowth of the author's long experience as a classroom teacher and as one who has had charge of teacher training at the University of Pennsylvania. It represents his philosophy and experience and should be read by all secondary school teachers of mathematics.

Dean Minnick is not concerned with the presentation of specific methods of meeting each educational problem, but rather in presenting and illustrating broad educational principles in a way that will make their applications clear. Moreover, he has not attempted to discuss all of the important topics in the field of secondary school mathematics, as a glance at the Table of Contents will reveal. There are eighteen chapters as follows:

I. The Evolution of Mathematics in the American High School
II. Some Principles of Teaching
III. The Aim of Mathematics Education
IV. Applications of Mathematics
V. The Course of Study
VI. Organization of Materials in Junior High School Mathematics
VII. Methods of Teaching Various Topics in Algebra
VIII. Content of the Course in Geometry
IX. Intuition and Experimentation
X. The Introduction to the Study of Demonstrative Geometry
XI. Teaching the Pupil to Think in Geometry
XII. Definitions and Axioms
XIII. Solid Geometry
XIV. Theory of Limits
XV. Organization and Generalizations
XVI. The Textbook
XVII. Mathematical Tests
XVIII. Mathematics Clubs.

The book is timely as there is great need just now for a single treatise of this kind that will be provocative, but not final, and which will stimulate teachers to search the literature further for the answers to questions not answered by this particular text. The book should prove useful as a text in teacher training classes.

The book is well written, freely illustrated, and is bound in an attractive cover.

Extensive bibliographies at the end of each chapter make the book all the more desirable as a text.

W. D. R.

The New Progress Arithmetics. By Philip A. Boyer, W. Walker Cheyney, and Holman White. Books A to E inclusive. The Macmillan Company, 1940. Price, $.48.

These five new books form a variant edition of the first five books previously published under the title "Progress Arithmetics" to conform to a revised grade placement, which is becoming increasingly common in many localities, whereby many of the more formidable understandings and skills are undertaken from one to two years later than in the older and more conventional courses. Teachers who believe that such material should be postponed in this way will find these books of interest.

The books are attractive, well illustrated, simple, and yet they cover the field well.

W. D. R.

Elementary Mathematics from an Advanced Standpoint. Geometry. By Felix Klein. Translated from the third German edition by E. R. Hedrick and C. A. Noble. The Macmillan Company, 1939. ix+214 pp. Price, $3.50.

Students of mathematics who are familiar with the English translation of Klein's Volume I will be interested in this book which is Volume II of his three-volume work.

The book is divided into three parts. Part I deals with "The Simplest Geometric Manifolds," Part II with "Geometric Transformation," and

Part III with the "Systematic Discussion of Geometry and Its Foundations."

The book deals with the historical aspects of various concepts and should be useful to teachers by helping them better to arrange and classify geometric facts and to see the place of geometry in the field.

The book will be helpful both to teachers who are preparing to teach and to experienced teachers of geometry.

W. D. R.

The Training of Mathematics Teachers for Secondary Schools in England and Wales and in the United States. By Ivan Stewart Turner, Fourteenth Yearbook of the National Council of Teachers of Mathematics, 1939. (New York: The Bureau of Publications, Teachers College, Columbia University). xiii+231 pp. Price, $1.25.

Secondary school mathematics in the United States is passing through a critical period. Fewer students each year are being enrolled in mathematics courses; fewer courses are being taught; and many of the students who take the subject complain about its dullness, its difficulty, and its lack of meaning to them. Some extremists are already predicting that the day of mathematics is past—that the subject will go the way of Latin.

But the "Queen of the Sciences," the indispensable tool of industry with its precision and research, and the logical discipline with a rich cultural heritage has something essential to contribute to every boy and girl. Because the subject is not fulfilling its promise, it must be the teachers and the teaching that are responsible. How should the teacher training program be changed to meet the needs of prospective teachers who are to do a good job of stimulating teaching?

In England and Wales the pupils who go on with their secondary school mathematics learn it well, albeit in a formal and stilted setting by instructors whose academic preparation had its emphasis on subject matter. In America the kind of mathematics we teach is more diversified and flexible, in many places taught by methods characterized as "progressive," by persons whose college training has stressed professional aspects. But in general we are getting poorer results in America, although it must be admitted that we are trying to teach mathematics to many students who would not be permitted to study it in England.

But perhaps a comparison of the kind of mathematics taught in England and Wales and in America, with an examination of the professional and academic training undergone by teachers in these countries, and a study of the methods employed will give a clue to better mathematics teaching. Dr. Turner has made these comparisons with the thesis that for prospective teachers who already possess desirable personal and scholastic requirements "there is a place of considerable importance for formal professional education." He has given answers to a number of relevant questions which are here stated verbatim from his work.

1. How much mathematics is studied by secondary school pupils in the two countries, and what standard is achieved?

2. Is the academic training of the mathematics teachers adequate to guarantee reasonably good teaching of school mathematics at the standard required in the schools?

3. What is the nature of and how effective are the present methods of professional training of mathematics teachers in the two countries?

4. What facilities are provided for in-service training of mathematics teachers? How effective are these facilities?

5. What agencies other than the regular teacher training institutions contribute to the training of mathematics teachers, and what is the nature and importance of their contribution?

6. What assumptions are implicit in the methods of training adopted in the two countries?

7. On what principles should the methods of training be based? In the light of these principles what are the strengths and weaknesses of the training of mathematics teachers in England and Wales and in the United States?

Dr. Turner's technique in his investigation consisted in defining nine principles which are fundamental to the training of mathematics teachers. These, then, provide the criteria in terms of which the present methods of training mathematics teachers in England and the United States can be evaluated. The influences of texts, examining bodies, professional agencies, universities and colleges, and certification requirements are discussed. Examples of mathematics examinations in pure and professional mathematics courses are given. Fifteen tables provide data on numbers of pupils and teachers in mathematics, examination subjects, prescribed and elective courses, and qualification of teachers in these countries.

The significance of Dr. Turner's work lies in the answers he has given to the questions stated above. He reveals the strong and weak points of teacher training programs in continental England and the United States in a comparative setting which will aid each of the two countries in reorganizing their present practices.

A. E. KATRA